INVENTORY COST EFFECT OF CONSOLIDATING SEVERAL ONE-WAREHOUSE MULTIRETAILER SYSTEMS

WEI-SHI LIM
Department of Marketing, NUS Business School, National University of Singapore, weishi@nus.edu.sg

JIHONG OU and CHUNG-PIAW TEO
Department of Decision Sciences, NUS Business School, National University of Singapore
bizoujh@nus.edu.sg • bizteocp@nus.edu.sg

Consolidation of warehouses is a new trend in global logistics management, and the reduction in order processing and inventory costs is often cited as one of the main motivations. In this note we show that when retailers face constant demand rates and their ordering costs are independent of the warehouse that services them, consolidated systems are *rarely* suboptimal and *always* lead to close-to-optimal inventory replenishment costs. In particular, we prove that using two (one) properly selected warehouses, the systemwide inventory replenishment cost is in the worst case at most 2% (14.75%) more than the optimal.

Received April 2001; revisions received February 2002, August 2002; accepted October 2002.
Subject classifications: Inventory/production: multi-item/echelon/stage, deterministic. Facilities/equipment planning: location, discrete.
Area of review: Manufacturing, Service, and Supply Chain Operations.

1. INTRODUCTION

Many companies are streamlining their distribution network by consolidating and centralizing their logistics operations. The Strategies 2005 Report prepared by leading food distributors in the United States suggested that warehouse consolidation is a key component of the logistic strategy in the industry (*Food Distributor* 2000). Consolidation strategy is also often pursued with great enthusiasm (Weiskott 1998) in cases in which the products are of high value and low weight type (as in most electronics components), and when transportation costs are insignificant compared to inventory and ordering costs.

Our intent in this note is to examine the benefits of consolidation on inventory replenishment cost. Given the popularity of the consolidation strategy and the common belief that consolidation helps to reduce systemwide inventory cost resulting from risk pooling and economies of scale, we view that it is timely to scrutinize this belief based on a quantitative model. In a single-echelon system with deterministic demand in which holding and ordering costs at the retailers are not considered, it is easy to see why consolidation is optimal. In fact, in this case each warehouse acts as a single-stage EOQ system, and it is well known that the average inventory replenishment cost at each warehouse is concave in the demand assigned. Exploiting concavity property, it is easy to see that the optimal strategy is to consolidate all demand at a single warehouse.

We show in this note that the same qualitative insight actually carries over to a two-echelon system in which both the warehouses and retailers have inventory holding and ordering costs. In a stable demand environment, where the retailer ordering costs are independent of the warehouse that services them, we show that consolidated systems are *rarely* suboptimal and *always* lead to close-to-optimal inventory replenishment costs. In particular, we prove that using two (one) properly selected warehouses, the systemwide inventory replenishment cost is in the worst case at most 2% (14.75%) more than the optimal. So the suggestion to managers facing consolidation issues with stable demand is that savings in inventory replenishment cost can often be expected, and the spotlight should thus be on the transportation costs.

We want to caution that the conclusion is reached without consideration of the impact on the operating costs in order processing, transportation, etc. We would imagine consolidation might lead to increased economies of scale in purchasing and transportation operations, but unit transportation costs could increase because the goods are now shipped over a longer distance. On another front, Teo et al. (2001), building on the work of Gallego (1998) on inventory replenishment cost approximation and bounds for (r, Q) systems, found that if the retailers face random demands, even in a single-echelon system, consolidating warehouses of different characteristics may lead to suboptimal performance and in fact may deviate very far from the optimal. Teo and Shu (2001) have also recently built upon the results in this note by devising a column generation algorithm to solve the general distribution network design problem with both inventory and transportation cost incorporated.

One-warehouse multiretailer systems with deterministic customer demands have been studied extensively since the breakthrough work of Roundy (1985) (see the review paper of Muckstadt and Roundy 1993). The approach to proving the note's main results centers around Roundy's lower bound for a single one-warehouse multiretailer inventory system. Let the holding cost rate and the fixed charge at the warehouse be $2h_0$ and K_0, and at retailer i, $i=1,\ldots,N$, be $2h_i$ and K_i, and the demand rate at retailer i be λ_i. Take the standard assumption that $h_i > h_0$ for every $i=1,\ldots,N$. An inventory control policy for the system can be characterized by an $N+1$-tuple, $(T_0, T_1, \ldots, T_N)$, where T_0 is the reorder interval at the warehouse and T_i is that at retailer i, $i=1,\ldots,N$. The 98% optimal lower bound obtained by Roundy (1985) is described precisely in the following proposition.

Proposition 1. (i) *The solution to the following convex optimization problem,*

$$\min_{T_i>0,\, i=0,1,\ldots,N} \frac{K_0}{T_0}+\sum_i \frac{K_i}{T_i} + \lambda_i h_i T_i + \lambda_i h_0[\max(T_0,T_i)-T_i], \tag{1}$$

is a lower bound on the average cost of any feasible control policy, and the solution can be rounded off to obtain a feasible integer-ratio policy with a cost within 2% *of the minimum of* (1).

(ii) *In the solution to* (1), *the retailers can be divided into three groups*: *G*, *L*, *and E*. *For the retailers in G*, *their reorder interval T_i is given by*

$$T_i=\sqrt{K_i/\lambda_i h_i} > T_0;$$

for the retailers in L, *their reorder interval is given by*

$$T_i=\sqrt{K_i/\lambda_i(h_i-h_0)} < T_0;$$

and for the retailers in E, *their reorder interval is the same as that at the warehouse and is given by*

$$T_i=T_0=\sqrt{\left[K_0+\sum_{i\in E}K_i\right]\Big/\left[\sum_{i\in E}\lambda_i h_i+\sum_{i\in L}\lambda_i h_0\right]}.$$

Furthermore,

$$\sqrt{K_i/\lambda_i(h_i-h_0)} \geqslant T_0 \geqslant \sqrt{K_i/\lambda_i h_i} \quad \text{for all } i\in E. \tag{2}$$

Based on the solution structure as stated in Proposition 1, we deduce another property of the optimal solution to (1), $(T_0, T_1, \ldots, T_N)$, as

$$\begin{aligned}
&\frac{K_0}{T_0}+\sum_i\left(\frac{K_i}{T_i}+\lambda_i h_i T_i+\lambda_i h_0[\max(T_0,T_i)-T_i]\right)\\
&\quad=\sum_{i\in L}\left(\frac{K_i}{T_i}+\lambda_i(h_i-h_0)T_i\right)+\sum_{i\in G}\left(\frac{K_i}{T_i}+\lambda_i h_i T_i\right)\\
&\qquad+\frac{K_0}{T_0}+\sum_{i\in E}\left(\frac{K_i}{T_0}+\lambda_i h_i T_0\right)+\sum_{i\in L}\lambda_i h_0 T_0\\
&\quad=\sum_{i\in L}2\lambda_i(h_i-h_0)\sqrt{\frac{K_i}{\lambda_i(h_i-h_0)}}+\sum_{i\in G}2\lambda_i h_i\sqrt{\frac{K_i}{\lambda_i h_i}}\\
&\qquad+\sum_{i\in E}2\lambda_i h_i T_0+2\sum_{i\in L}\lambda_i h_0 T_0\\
&\quad\geqslant 2\sum_{i\in L}\lambda_i h_0 T_0+\sum_{i\in G}2\lambda_i h_i T_0\\
&\qquad+\sum_{i\in E}2\lambda_i h_i T_0 \geqslant 2\sum_{i=1}^{N}\lambda_i h_0 T_0.
\end{aligned} \tag{3}$$

These properties will be used later to show the following for many one-warehouse multi-retailer systems:

- If all retailers with reordering cycles greater than or equal to the reordering cycle of their own warehouse are moved to a warehouse with a shorter reordering cycle, the lower bound will not increase.
- If warehouses have only retailers with shorter than their own reordering cycles, consolidating all retailers in one of them will not increase the lower bound.
- In general, consolidating two warehouses will not increase the lower bound more than a factor of 1/9.

The above statements, proved exactly in the next section, lead to the conclusion of near optimality of consolidation.

2. THE MAIN RESULT

Now suppose there are $M \geqslant 2$ such one-warehouse multiretailer systems indexed by $j=1,\ldots,M$. In system j, there are N_j retailers (grouped in set I_j), whose ordering interval, demand rates, holding cost rates, and fixed charge are $T_{j,i}$, $\lambda_{j,i}$, $2h_{j,i}$, and $K_{j,i}$, for $i=1,\ldots,N_j$, respectively; and at the warehouse in system j, the ordering interval, holding cost rate, and the fixed charge are $T_{j,0}$, $2h_{j,0}$, and $K_{j,0}$. We note here that all the parameters (including the ordering cost of the retailer $K_{j,i}$) are constants that do not depend on which warehouse is serving the retailer.

Denote the minimal inventory replenishment cost for system j by C_j^*, $j=1,\ldots,M$. Then, the optimum systemwide inventory replenishment cost under the separate systems is $C^*=\sum_{j=1}^{M}C_j^*$.

We first prove our main result for the case of $M=2$. Given a set of retailers R, define

$$\begin{aligned}C_1(R)=\min_{T_{1,0},\,T_{1,i},\,i\in R}\ &\frac{K_{1,0}}{T_{1,0}}+\sum_{i\in R}\frac{K_{1,i}}{T_{1,i}}+\lambda_{1,i}h_{1,i}T_{1,i}\\
&+\lambda_{1,i}h_{1,0}[\max(T_{1,0},T_{1,i})-T_{1,i}],\end{aligned}$$

and

$$\begin{aligned}C_2(R)=\min_{T_{2,0},\,T_{2,i},\,i\in R}\ &\frac{K_{2,0}}{T_{2,0}}+\sum_{i\in R}\frac{K_{2,i}}{T_{2,i}}+\lambda_{2,i}h_{2,i}T_{2,i}\\
&+\lambda_{2,i}h_{2,0}[\max(T_{2,0},T_{2,i})-T_{2,i}].\end{aligned}$$

Then, by Proposition 1, $C_1(I_1)$ is a lower bound to C_1^*, $C_2(I_2)$ is a lower bound to C_2^*, $C_1(I_1\cup I_2)$ is a lower bound to the consolidated system with warehouse 1 serving all the retailers, and $C_2(I_1 \cup I_2)$ is a lower bound to the consolidated system with warehouse 2 serving all the retailers. If the integer ratio policy derived from the solution to $C_j(I_1 \cup I_2)$ is used to control the consolidated system at warehouse j, $j = 1, 2$, the following theorem leads to the conclusion that one of the two consolidated systems will achieve a cost within $(1.02 \times 9/8 - 1) \times 100\% = 14.75\%$ of $C_1(I_1) + C_2(I_2)$, which is a lower bound to $C_1^* + C_2^* = C^*$.

THEOREM.

$$\min(C_1(I_1 \cup I_2), C_2(I_1 \cup I_2)) \leqslant \frac{9}{8}\min_R[C_1(R) + C_2(I_1 \cup I_2 - R)] \leqslant \frac{9}{8}[C_1(I_1) + C_2(I_2)].$$

PROOF. We just need to prove the first inequality, and for that, assume $(R_1, R_2 = I_1 \cup I_2 - R_1)$ minimizes $C_1(R) + C_2(I_1 \cup I_2 - R)$. To simplify the notation, we denote the optimal solution for $C_1(R_1)$ $(C_2(R_2))$ by $\{T_{1,0}, T_{1,i}: i \in R_1\}$ $(\{T_{2,0}, T_{2,i}: i \in R_2\})$. By Proposition 1, in the solution R_j splits into three groups: G_j, L_j, and E_j, $j = 1, 2$. Suppose WLOG $T_{1,0} \leqslant T_{2,0}$. If it is also the case that $h_{1,0} \leqslant h_{2,0}$, we can just reassign all the retailers in R_2 to warehouse 1, and their costs would be lower than when served by warehouse 2. Thus, we have $C_1(I_1 \cup I_2) \leqslant C_1(R_1) + C_2(R_2)$, and the theorem is proved. So we consider the nontrivial case in which

$$h_{1,0} > h_{2,0}. \tag{4}$$

We claim also that we can assume WLOG

$$T_{2,i} < T_{1,0} \quad \text{for all } i \in R_2. \tag{5}$$

In particular, we have $G_2 = \emptyset$ and $E_2 = \emptyset$. Otherwise, if there is a retailer $i \in R_2$ such that $T_{2,i} \geqslant T_{1,0}$, we reassign it to warehouse 1 and take its order interval still as $T_{2,i}$. Note that the ordering cost of warehouse 1 does not change because the ordering interval remains the same. The cost associated with this retailer in the new assignment is $K_{2,i}/T_{2,i} + \lambda_{2,i}h_{2,i}T_{2,i} + \lambda_{2,i}h_{1,0}[\max(T_{1,0}, T_{2,i}) - T_{2,i}] = K_{2,i}/T_{2,i} + \lambda_{2,i}h_{2,i}T_{2,i}$, which is no larger than that in the original assignment, $K_{2,i}/T_{2,i} + \lambda_{2,i}h_{2,i}T_{2,i} + \lambda_{2,i}h_{2,0}[\max(T_{2,0}, T_{2,i}) - T_{2,i}]$. Because the costs associated with all other retailers are unchanged, the new assignment gives another solution that is at least as good as (R_1, R_2) in minimizing $C_1(R) + C_2(I_1 \cup I_2 - R)$ and it does have property (5). If the ordering intervals constructed above are not optimal under the new assignment (because the optimal warehouse ordering intervals may have changed under the new assignment), we have found a new warehouse-retailer assignment with smaller total inventory cost compared to (R_1, R_2). This gives rise to a contradiction because (R_1, R_2) is an optimal solution.

To prove the first inequality in the theorem, we take a feasible solution for $C_1(I_1 \cup I_2)$ as $\{T_{1,0}, T_{1,i}: i \in R_1, T_{2,i}: i \in R_2\}$. Its cost is

$$\begin{aligned}
&\frac{K_{1,0}}{T_{1,0}} + \sum_{i \in R_1}\left\{\frac{K_{1,i}}{T_{1,i}} + \lambda_{1,i}h_{1,i}T_{1,i} + \lambda_{1,i}h_{1,0}[\max(T_{1,0}, T_{1,i}) - T_{1,i}]\right\} \\
&\quad + \sum_{i \in R_2}\left\{\frac{K_{2,i}}{T_{2,i}} + \lambda_{2,i}h_{2,i}T_{2,i} + \lambda_{2,i}h_{1,0}[\max(T_{1,0}, T_{2,i}) - T_{2,i}]\right\} \\
&= C_1(R_1) + C_2(R_2) + \sum_{i \in R_2}\lambda_{2,i}h_{1,0}[\max(T_{1,0}, T_{2,i})] - \frac{K_{2,0}}{T_{2,0}} \\
&\quad - \sum_{i \in R_2}\lambda_{2,i}(h_{2,0}[T_{2,0} - T_{2,i}] + h_{1,0}T_{2,i}) \\
&\leqslant C_1(R_1) + C_2(R_2) + \sum_{i \in R_2}\lambda_{2,i}h_{1,0}T_{1,0} - \sum_{i \in R_2}\lambda_{2,i}h_{2,0}T_{2,0} - \frac{K_{2,0}}{T_{2,0}}.
\end{aligned}$$

The last inequality is due to $h_{1,0} > h_{2,0}$. Note that because $G_2 \cup E_2 = \emptyset$, $T_{2,0} = \sqrt{K_{2,0}/[\sum_{i \in R_2}\lambda_{2,i}h_{2,0}]}$, or equivalently, $K_{2,0}/T_{2,0} = \sum_{i \in R_2}\lambda_{2,i}h_{2,0}T_{2,0}$. We obtain

$$C_1(I_1 \cup I_2) \leqslant C_1(R_1) + C_2(R_2) + \left(\frac{\Lambda_2}{\Lambda_1}\right)\Lambda_1 h_{1,0}T_{1,0} - 2\Lambda_2 h_{2,0}T_{2,0}, \tag{6}$$

where

$$\Lambda_1 = \sum_{i \in R_1}\lambda_{1,i}, \quad \Lambda_2 = \sum_{i \in R_2}\lambda_{2,i}.$$

Next, take a feasible solution for $C_2(I_1 \cup I_2)$ as $\{T_{2,0}, T_{1,i}: i \in R_1, T_{2,i}: i \in R_2\}$. We can obtain

$$\begin{aligned}
&\frac{K_{2,0}}{T_{2,0}} + \sum_{i \in R_1}\left\{\frac{K_{1,i}}{T_{1,i}} + \lambda_{1,i}h_{1,i}T_{1,i} + \lambda_{1,i}h_{2,0}[\max(T_{2,0}, T_{1,i}) - T_{1,i}]\right\} \\
&\quad + \sum_{i \in R_2}\left\{\frac{K_{2,i}}{T_{2,i}} + \lambda_{2,i}h_{2,i}T_{2,i} + \lambda_{2,i}h_{2,0}[\max(T_{2,0}, T_{2,i}) - T_{2,i}]\right\} \\
&= C_1(R_1) + C_2(R_2) + \sum_{i \in R_1}\lambda_{1,i}h_{2,0}[\max(T_{2,0}, T_{1,i}) - T_{1,i}] \\
&\quad - \frac{K_{1,0}}{T_{1,0}} - \sum_{i \in R_1}\lambda_{1,i}h_{1,0}[\max(T_{1,0}, T_{1,i}) - T_{1,i}] \\
&\leqslant C_1(R_1) + C_2(R_2) + \sum_{i \in R_1}\lambda_{1,i}h_{2,0}[T_{2,0}] - \frac{K_{1,0}}{T_{1,0}}.
\end{aligned}$$

Thus, we have

$$C_2(I_1 \cup I_2) \leqslant C_1(R_1) + C_2(R_2) + \left(\frac{\Lambda_1}{\Lambda_2}\right)\Lambda_2 h_{2,0}T_{2,0}. \tag{7}$$

Combining (6) and (7) and choosing two positive numbers α and β, we have

$$\begin{aligned}\alpha C_1(I_1 \cup I_2) + \beta C_2(I_1 \cup I_2) \\ \leqslant (\alpha+\beta)C_1(R_1) + (\alpha+\beta)C_2(R_2) \\ + \alpha\left[\frac{\Lambda_2}{\Lambda_1}\Lambda_1 h_{1,0}T_{1,0} - 2\Lambda_2 h_{2,0}T_{2,0}\right] \\ + \beta\left[\frac{\Lambda_1}{\Lambda_2}\Lambda_2 h_{2,0}T_{2,0}\right].\end{aligned} \quad (8)$$

Choosing

$$\alpha = \Lambda_1^2/(\Lambda_1+\lambda_2)^2, \quad \beta = 1-\alpha,$$

we can simplify (8) into

$$\begin{aligned}\alpha C_1(I_1 \cup I_2) + \beta C(I_1 \cup I_2) \\ \leqslant C_1(R_1) + C_2(R_2) \\ + \frac{\Lambda_1\Lambda_2}{(\Lambda_1+\Lambda_2)^2}(\Lambda_1 h_{1,0}T_{1,0} + \Lambda_2 h_{2,0}T_{2,0}).\end{aligned}$$

By (3), we have

$$C_1(R_1) \geqslant 2\Lambda_1 h_{1,0}T_{1,0} \quad \text{and} \quad C_2(R_2) \geqslant 2\Lambda_2 h_{2,0}T_{2,0}.$$

So

$$\begin{aligned}\alpha C_1(I_1 \cup I_2) + \beta C(I_1 \cup I_2) \\ \leqslant \left(1 + \frac{1}{2}\frac{\Lambda_1\Lambda_2}{(\Lambda_1+\Lambda_2)^2}\right)[C_1(R_1) + C_2(R_2)].\end{aligned}$$

Note that

$$1 + \frac{1}{2}\frac{\Lambda_1\Lambda_2}{(\Lambda_1+\Lambda_2)^2} \leqslant 1 + \frac{1}{8},$$

and so the theorem follows. □

Note that inequality (7) is rather loose. This arises because we are not able to impose a more refined structure on the optimal ordering intervals in the set R_1. If in the above proof, both $G_1 \cup E_1$ and $G_2 \cup E_2$ happen to be empty, the arguments can be tightened to obtain the following stronger result.

PROPOSITION 2. *In the case when $G_1 \cup E_1 = \emptyset$ and $G_2 \cup E_2 = \emptyset$,*

$$\begin{aligned}\min(C_1(I_1 \cup I_2), C_2(I_1 \cup I_2)) \leqslant \min_R[C_1(R) + C_2(I_1 \cup I_2 - R)] \\ \leqslant [C_1(I_1) + C_2(I_2)].\end{aligned} \quad (9)$$

PROOF. To see this, note that we can now improve (7) to

$$\begin{aligned}C_2(I_1 \cup I_2) &\leqslant C_1(R_1) + C_2(R_2) + \left(\frac{\Lambda_1}{\Lambda_2}\right)\Lambda_2 h_{2,0}T_{2,0} - \frac{K_{1,0}}{T_{1,0}} \\ &= C_1(R_1) + C_2(R_2) + \left(\frac{\Lambda_1}{\Lambda_2}\right)\Lambda_2 h_{2,0}T_{2,0} \\ &\quad - \Lambda_1 h_{1,0}T_{1,0}.\end{aligned} \quad (10)$$

The last equality follows from the fact that now $L_1 = R_1$. Because either $h_{1,0}T_{1,0} \leqslant 2h_{2,0}T_{2,0}$ or $h_{2,0}T_{2,0} \leqslant h_{1,0}T_{1,0}$, we have either

$$\Lambda_2 h_{1,0}T_{1,0} - 2\Lambda_2 h_{2,0}T_{2,0} \leqslant 0$$

or

$$\Lambda_1 h_{2,0}T_{2,0} - \Lambda_1 h_{1,0}T_{1,0} \leqslant 0.$$

The result follows from (6) and (10). □

Now we can extend the theorem to the case of $M > 2$. Suppose for the *optimal* separate systems, warehouse 1 has the shortest ordering cycle, i.e., $T_{1,0} \leqslant T_{j,0}$ for all $j > 1$. Then, using the same arguments as in the proof of the theorem, we can again claim that

$$T_{j,i} < T_{1,0} \leqslant T_{j,0} \quad \text{for all } i \in I_j,\ j > 1. \quad (11)$$

In particular, we have $G_j \cup E_j = \emptyset$ for $j > 1$. Thus, by (9), all the systems other than system 1 can be consolidated to a single one-warehouse system. Let's say they are all consolidated to warehouse 2. Then, we have

$$C_2(I_2 \cup \cdots \cup I_M) \leqslant \sum_{j=2}^{M} C_j(I_j),$$

which implies the following.

COROLLARY 1. *The lower bound to two separate systems with system 1 serving customers in I_1 and system 2 serving all the other customers is a lower bound for the inventory replenishment cost function in the separate M systems.*

COROLLARY 2. *If consolidating further to a single system, the total inventory replenishment cost is within 14.75% of C^*. Specifically, we have*

$$\min(C_1(I_1 \cup \cdots \cup I_M), C_2(I_1 \cup \cdots \cup I_M)) \leqslant \frac{9}{8}\sum_{j=1}^{M} C_j(I_j).$$

Nonoptimality Example

We have performed an extensive numerical study to evaluate empirically the effect of consolidation. For over 10,000 separate systems, consolidation into a single-warehouse system was always found to be optimal. With considerable effort and luck, we managed to find an example with two separate inventory systems for which consolidating into a single system leads to higher cost (Table 1). Note that $C_1(I_1) = 216.7876571$ and $C_2(I_2) = 23.75070158$.

The two warehouses have vastly different cost structures: Warehouse 1, with low ordering and high holding cost, is ideal for cross-docking of inventory to the retailers. Warehouse 2, on the other hand, is ideal for staging of inventory,

Table 1.

	Ordering Cost	Holding Cost	Demand
Warehouse 1 serving 3 retailers:			
Warehouse 1	$K_{1,0} = 0.19192444$	$h_{1,0} = 145.735398$	
Retailer 1	$K_{1,1} = 2.38191032$	$h_{1,1} = 145.7354885$	$\lambda_{1,1} = 120.1545886$
Retailer 2	$K_{1,2} = 0.009152176$	$h_{1,2} = 148.5858742$	$\lambda_{1,2} = 0.419055781$
Retailer 3	$K_{1,3} = 0.045295548$	$h_{1,3} = 145.7353995$	$\lambda_{1,3} = 2.114537948$
Warehouse 2 serving 1 retailer:			
Warehouse 2	$K_{2,0} = 68.4471941$	$h_{2,0} = 0.1350449$	
Retailer 4	$K_{2,1} = 0.000001$	$h_{2,1} = 145.73616$	$\lambda_{2,1} = 60.54503063$

especially for those retailers with high holding cost. Thus, it is ideal to use warehouse 2 to serve retailer 4.

If we were to use warehouse 1 to serve retailer 4, then it needs to synchronize its replenishment activities to support the frequent shipments to retailer 4. This will affect the cost it incurred to serve the other retailers. In this case, $C_1(I_1 \cup I_2) = 249.8709473$. Similarly, consolidating all retailers at warehouse 2 is not cost effective, because the cost of staging inventory at the warehouse increases with the larger demand now served by the warehouse. Consequently, $C_2(I_1 \cup I_2) = 249.8709475$.

Note that

$$1.0388 \times (C_1(I_1) + C_2(I_2)) = \min(C_1(I_1 \cup I_2), C_2(I_1 \cup I_2)).$$

Because we know that the convex relaxation is a valid lower bound and within 2% of the optimal inventory replenishment costs, we obtain an example for which consolidation to a single system is worse off than the separate systems.

The deviation from optimality is just 1.84% more than that of the separate ones, far from the theoretical worst-case bound of 14.75% presented in this note. It is an interesting problem to find out whether the gap between the worst case bound and the worst case example can be narrowed.

ACKNOWLEDGMENTS

The authors thank Prof. Federgruen, the associate editor, and two referees for their comments and suggestions that led to an improved version of this note. This research was supported by NUS research grant R-314-000-030-112, and a fellowship from the Singapore-MIT Alliance Program in High Performance Computation for Engineered System.

REFERENCES

Food Distributor. 2000. (Mar./Apr.).

Gallego, G. 1998. New bounds and heuristics for (Q, r) policies. *Management Sci.* **44** 219–233.

Muckstadt, J. A., R. O. Roundy. 1993. Analysis of multistage production systems. S. C. Graves, A. H. G. Rinooy Kan, P. H. Zipkin, eds. *Logistics of Production and Inventory*. North-Holland, Amsterdam, The Netherlands, 59–131.

Roundy, R. O. 1985. 98% effective integer-ratio lot-sizing for one warehouse multi-retailer systems. *Management Sci.* **31** 1416–1430.

Teo, C. P., J. Shu. 2001. Warehouse-retailer network design problem. *Oper. Res.* Forthcoming.

——, J. H. Ou, K. H. Goh. 2001. Impact on inventory costs with consolidation of distribution centers. *IIE Trans.* **33**(2) 99–110.

Weiskott, M. N. 1998. Consolidate or scatter? *Plants Sites Parks* (Oct./Nov.).