American Economic Review 2015, 105(7): 2120–2140
http://dx.doi.org/10.1257/aer.20121469

Team Contests with Multiple Pairwise Battles†

By Qiang Fu, Jingfeng Lu, and Yue Pan*

We consider a multi-battle team contest in which players from two rival teams form pairwise matches to fight in distinct component battles, which are carried out sequentially or (partially) simultaneously. A team wins if and only if its players win a majority of battles. Each player benefits from his team's win, while he can also receive a private reward for winning his own battle. We find that the outcomes of past battles do not distort the outcomes of future battles. Neither the total expected effort nor the overall outcome of the contest depends on the contest's temporal structure or its feedback policy. (*JEL* C72, D72, D74, D82)

Many competitive situations require that contenders meet each other on multiple fronts, and the winner is determined by aggregating the parties' overall performance in the entire series of confrontations (see Konrad 2009). These competitions often take place between adversarial teams, alliances, or groups as coalitions of affiliated but independent players who share social, political, or commercial interests. As intuitively exemplified by many sports events with team titles, players from rival teams form pairwise matches and compete head-to-head; individual players take decentralized actions to vie for victories on their own battlefields, while their advance contributes to their team's win.[1] We label such contentions "team contests

* Fu: Department of Strategy and Policy, National University of Singapore, 15 Kent Ridge Drive, Singapore, 119245 (e-mail: bizfq@nus.edu.sg); Lu: Department of Economics, National University of Singapore, 1 Arts Link, Singapore, 117570 (e-mail: ecsljf@nus.edu.sg); Pan: Department of Economics and Department of Chemical Engineering, National University of Singapore, 117570 (e-mail: delia_py@hotmail.com). We are grateful to three anonymous reviewers for very thoughtful and detailed advice on revising our paper. We are grateful to Kai Konrad for an inspiring and encouraging discussion. We thank Michael Baye, Carmen Beviá, Yan Chen, John Conley, Luis Corchón, Andrew Daughety, Jeffrey Ely, Yuk-Fai Fong, Ganesh Iyer, Tilman Klumpp, Dan Kovenock, Roger Lagunoff, Wolfgang Leninger, Wooyoung Lim, Ed Malesky, David Malueg, Jennifer Reinganum, Brian Roberson, Ron Siegel, Yasutora Watanabe, Quan Wen, Myrna Wooders, and audiences at the 2012 Spring Midwest Economic Theory Conference (Indiana University); the 2012 Society for the Advancement of Economic Theory Conference (Queensland University); and the economics departmental seminar at Hong Kong University of Science and Technology, Shanghai University of Finance and Economics, and Vanderbilt University for helpful comments and suggestions. The usual disclaimer applies. Qiang Fu thanks the Max Planck Institute of Public Finance and Tax Law for its support and hospitality; part of the project was executed when Fu visited the institute. Jingfeng Lu is grateful to Vanderbilt University, Ohio State University, and Northwestern University for kindly hosting his sabbatical leave visits. Fu and Lu gratefully acknowledge the financial support from Ministry of Education, the Republic of Singapore. We, the authors, declare that we have no relevant or material financial interests that relate to the research described in this paper.
† Go to http://dx.doi.org/10.1257/aer.20121469 to visit the article page for additional materials and author disclosure statement(s).

[1] Consider, for instance, the Ryder Cup in men's and women's golf, the Davis Cup and the Fed Cup in men's and women's tennis, the Thomas Cup and the Uber Cup in men's and women's badminton, and the Swaythling Cup and the Corbillon Cup in men's and women's table tennis.

with multiple pairwise battles." In this paper, we conduct a benchmark theoretical analysis of such contests.

More saliently, various collective decision-making processes resemble such a contest. Consider, for instance, the general elections in most democracies, such as the House of Representatives and Senate elections in the United States: candidates representing rival parties compete for legislative seats in each constituency, and a party can form a government or set political agenda in the legislature if it acquires majority status.[2] Furthermore, general elections in many democracies are contested between political alliances as coalitions of individual political parties: parties form a bloc to pursue collective electoral success, while adopting independent strategies in campaigns.[3] For instance, India's electoral politics have been dominated by the contention between the National Democratic Alliance and the United Progressive Alliance, which are led, respectively, by two major national parties, the Bharatiya Janata Party (BJP) and the Indian National Congress. Both of the two leading national parties require the success of allied regional parties in specific states to clear electoral thresholds.

Collective contests are not uncommon in industry. Harris and Vickers (1987) contend that an R&D race for an innovative product is a continuous competition on a series of component technologies; the ultimate success requires that one party make a sufficient number of advances ahead of its competitors. Extensive evidence has demonstrated that interfirm R&D collaboration has become increasingly popular: firms pool complementary resources or expertise to form cross-functional alliances; competitions take place between alliances. For instance, major pharmaceutical and biotechnology companies, such as Roche and L'Oréal, have evolved into network organizations that rely heavily on extensive webs of strategic alliances in R&D. A project—e.g., the development of a new drug—is split and sourced to member entities within each alliance.[4]

Two common features of such contests motivate our analysis. First, typical externalities arise among team members: A team's win benefits all its members, while each bears the cost of his own contributions. Team members are only linked indirectly, through the prospect of winning the competition. Centralized coordination, to facilitate cost sharing and internalize players' interests, is often absent or infeasible. Alliances or interest groups can develop informally, in which players coalesce into different blocs based solely on their interests, opinions, or ideological positions; one example is the recent ascent of anti-euro advocates across EU states in elections for the European Parliament. Even in formally organized activities, central coordination often plays only a limited role. As pointed out by Snyder (1989), and confirmed by recent statistics, political parties in US congressional races supply only a small fraction of candidates' total funding. A candidate's success depends

[2] Cox and Magar (1999) and Hartog and Monroe (2008) demonstrate empirically that a political party's majority status in a national legislative body is a valuable asset to individual politicians in the party and worth considerable collective effort to attain.

[3] See "The ACE Encyclopaedia: Parties and Candidates," Electoral Knowledge Network, http://www.aceproject.org (accessed June 24, 2014).

[4] In the famous Intel-Sony-Toshiba alliance to develop power chips for consumer electronics, Intel was responsible for chip development, Sony customized the chip to meet consumers' demands, and Toshiba specialized in manufacturing.

primarily on his/her own campaign input, which is often nonpecuniary, unverifiable, and nontransferable.[5]

Second, team members' efforts interact more subtly than in a typical team production. Instead of joining forces to produce a single output, each individual player carries out a separate task on behalf of his team, i.e., rivaling his matched opponent on one particular front. One's victory discretely increases the team's lead, while his input cannot directly substitute for that of his teammates, and vice versa. In a biparty election, for example, a lopsided victory in one constituency cannot make up for a party's marginal losses in another two. Further, in a cross-functional R&D alliance, each member focuses its expertise on one specific area.

Despite the broad commonalities, these scenarios are governed by diverse temporal structures. House of Representatives elections in the United States, as well as the typical national general elections, largely resemble a simultaneous contest, as all votes are cast on the same day. In contrast, (partial) dynamics are embedded in Senate elections, with roughly one-third of the seats up for re-election every other year. An R&D race, as argued by Harris and Vickers (1987), is inherently a dynamic process that requires innovators to undergo multiple phases of incremental advances in component technologies. Taking a broader perspective, the development of various policy or institutional paradigms can be viewed as continuous and long-lasting contentions between opposing ideological blocs. For instance, the recent struggle in European Parliament elections is only one phase in the long-lasting and ongoing contention between euroskepticism and European integration initiatives.

The diversity in contest dynamics motivates us to investigate the ramifications of different temporal structures for players' strategic behavior and contest performance. Klumpp and Polborn (2006) demonstrate that sequential elections between two individual candidates—e.g., presidential primaries—outperform a simultaneous election—e.g., a presidential election—because the former lowers campaign expenditures. The comparison remains intriguing in our environment, in which the competition takes place between teams—as in House and Senate elections—instead of individual players. Klumpp and Polborn's finding can be attributed to a distorting "strategic momentum effect" or "discouragement effect," as identified by Harris and Vickers (1987) in a two-firm R&D race model: one's (perhaps purely accidental) early lead would allow him to attain easy wins in the future, as it forces his lagging opponent to concede prematurely. Such dynamic linkage or path dependence has yet to be explored in contests between teams. It remains unanswered how a team's existing status, e.g., lead or head start, could affect players' incentives. In their two-player multi-battle contest model, Aldrich (1980) and Strumpf (2002) show that the outcome of a contest sensitively depends on the particular sequencing of heterogeneous battles. A similar question warrants examination in our context as well. A thorough analysis of these issues not only sheds light on the strategic nature of the so-far-understudied team contest game, but also yields useful implications for contest design and practical policy experimentation.

[5] Party contributions (including coordinated expenditures) comprised 6 percent of total campaign expenditures for Senate races in 2010, while donations and contributions from individuals and political action committees (PACs) accounted for the lion's share. Candidates' own funds accounted for only 13 percent. ("Campaign Funding Sources for House and Senate Candidates, 1984–2012," The Campaign Finance Institute, http://www.cfinst.org/pdf/vital/VitalStats_t8.pdf, accessed August 25, 2013.)

In our model, two teams consist of an equal (odd) number of players and compete for a common object, which is a public good for all players on a team. Players from rival teams form pairwise matches, and each pair competes head-to-head in a distinct component battle. Battles can be carried out either sequentially or (partially) simultaneously. Each of them is resolved by a winner-selection mechanism that is homogeneous of degree zero in players' efforts.[6] A team wins if and only if its players win a majority of the battles. Besides the object awarded to the winning team, individual players can also receive private rewards for winning their own battles. Both the team prize and the private reward are commonly valued by the matched players in a battle. Players' heterogeneity is reflected in their nonidentically distributed bidding costs. They choose their efforts independently to maximize their own payoffs. We obtain three neutrality results, which sharply contrast with the usual wisdom.

History Independence.—When battles are carried out sequentially, the outcomes of early battles do not distort the balance of future confrontations, provided that the battle carries a positive "prize spread"—i.e., when the winning team has not been decided or the battle has a positive private reward. The probability of a player winning his battle is determined by his and his matched opponent's innate abilities, and remains independent of the state of the contest—i.e., how much one team is leading or lagging behind its rival or how many additional wins each team will need to reach the finish line.

Sequence Independence.—Fix the matching of players and let each battle be contended between a fixed pair of players. We allow the sequence of these battles to be reshuffled. The probability of each team's winning the contest and each player's ex ante expected payoff are independent of the ordering of battles. Each battle yields a fixed amount of ex ante expected overall effort regardless.[7]

Temporal-Structure Independence.—Under fixed matching of players, the ex ante expected effort of a battle and its stochastic outcome would not vary regardless of the temporal arrangement of battle dynamics, being either completely sequential or (partially) simultaneous. Neither the stochastic outcome of the whole contest nor its ex ante expected effort depends on the prevailing temporal structure.

Two key observations lead to our findings. First, each battle is commonly valued by the two players involved. In our context, each player participates in only one battle, and future outlays will be borne by his teammates.[8] Therefore, besides the fixed private reward, a player is only concerned about how the outcome of the current battle will affect the team's winning odds—i.e., his chance of winning the team trophy—when deciding on his effort. An intriguing and critical reciprocity comes

[6] This requirement is satisfied by most commonly adopted contest technologies; please refer to Section IC for details.

[7] A given pair of players' effort distributions could change with battle sequence. The result, however, confirms that the order of battles does not affect the game's equilibrium outcomes on expected dimensions. For instance, every player's ex ante expected effort would be the same regardless of battle sequence. More details will be provided in Section IIB.

[8] Players' payoff structure in our context sharply contrasts with that in a dynamic race, in which the same two individuals compete in all component battles—e.g., presidential primaries. In that case, each player takes into account his future effort costs when deciding on current input.

into play: because the contest is to be won by one and only one team, one team's advance is the other's backslide. A battle's outcome affects the teams' winning prospects symmetrically, and matched players always equally value winning their battle. Second, with common valuation, the homogeneity-of-degree-zero contest technology yields equilibrium strategies that are linear in valuation. Players' winning odds in each battle are therefore independent of the state of the contest, provided that the battle has a positive prize spread for contenders.

This key finding lays a foundation for our neutrality results. Consider a simple (although limited) case in which all battles provide positive private rewards, such that they have positive prize spreads regardless. The contest boils down to a series of independent lotteries. Each team's ex ante odds of winning the contest is thus independent of sequencing arrangement or temporal structure. In addition, when normalizing the (commonly) valued team prize to one, the common value for winning a battle is the sum of the private reward and the change in the probability of winning the team prize, which will be caused by the outcome of the battle. When evaluated from an ex ante perspective, neither component depends on the sequencing arrangement or temporal structure.[9] The linearity of equilibrium strategies in common value further generates neutrality results for ex ante expected effort in both a given battle and the whole contest.[10] More details will be revealed as our analysis unfolds.

Our analysis reveals the distinct incentives created by the unique competition structure of such team contests, and illuminates the novel strategic implications of team production and collective action in contest environments. It should be noted, however, that our study is conducted in a relatively stylized setting as a benchmark theoretical analysis. This framework allows us to identify a number of additional strategic or behavioral elements which, as will be discussed later in the paper, could either strengthen or nullify predictions obtained from our baseline setting and inspire future work.

This paper primarily contributes to the literature on multi-battle contests, which dates back to Harris and Vickers (1987). In addition to the important contribution of Klumpp and Polborn (2006), Malueg and Yates (2010) demonstrate theoretically the distortion caused by early outcome in a dynamic best-of-three contest with symmetric players and test it empirically. Konrad and Kovenock (2009) provide a complete characterization of the unique subgame perfect equilibrium for a generalized sequential multi-battle contest between two players. They show that equilibrium efforts, as well as players' winning probabilities, can be non-monotonic in the closeness of the race when bidders are asymmetric. When a component battle awards a positive intermediate prize, even a large lead by one player does not fully discourage the other. In a later study, Konrad and Kovenock (2010) demonstrate that the aforementioned discouragement effect can also be attenuated by uncertainty in players' bidding costs. Snyder (1989) studies a simultaneous multi-battle contest in which two political parties, as individual decision makers, compete in parallel

[9] The second component—i.e., the change in the probability of winning the team prize—is simply the probability that the battle is pivotal to the contest. More details will be provided later in the paper.

[10] The argument does not extend to the more general setting that allows for zero private reward in a battle, as a battle's outcome can affect the future dynamic path of the contest. Our analysis nevertheless establishes that despite the nuance, the neutralities in sequence and temporal structure hold generally.

elections in multiple constituencies. Kovenock and Roberson (2009) extend Snyder by repeating the competition in another period and introducing an intertemporal interdependence: A player's win on one battlefield in the first period grants him a head start when competitors meet again on the same front. Our paper differs from these studies, however, as they typically assume that the same two individual players meet and compete against each other in all component battles. Our results are unique to the setting of competitions between coalitions of players, and thus have a different scope of applications.

The rest of the paper is organized as follows. Section I sets up a general model of a multi-battle contest between teams, and Section II conducts the main analysis. In Section III, we discuss the main implications of our results and illustrate several possible extensions and caveats. Section IV concludes the paper. All technical proofs are relegated to the Appendix.

I. Setup

Two teams, indexed by $i = A, B$, compete for an indivisible object. Each team consists of $2n + 1$ risk-neutral players. Each player on one team is matched to an opponent from the rival team; the two players compete head-to-head in a *component battle* on one disjoint battlefield. A team is awarded the object if and only if it accumulates at least $n + 1$ victories from the $2n + 1$ pairwise component battles.[11]

The battles can be carried out sequentially or (partially or completely) simultaneously. For analytical convenience, we begin with a setting in which component battles are carried out completely successively. Battles are indexed by $t = 1, 2, \ldots, 2n + 1$, i.e., their orders in a given unfolding sequence. A player on team i is indexed by $i(t)$ if he is assigned to the tth battle. In Section IIB, we will present a formal analysis that allows the temporal structure of battle dynamics to be fully flexible.

The *state* of the contest before battle t is carried out is summarized by a tuple (k_A, k_B) with $t = k_A + k_B + 1$, where k_i is the number of wins secured by team i. The state (k_A, k_B) is observed by players $A(t)$ and $B(t)$ before they sink their efforts. A component battle t is called *trivial* if $\max\{k_A, k_B\} \geq n + 1$, i.e., the winning team has been determined.[12]

A. *Payoffs*

Rewards for each player arise from two sources: a *team prize* and a *battle prize*. First, one benefits from his team's win. The prize for the winning team is a public good, and all players on the team equally value it. We normalize the common valuation to one. Second, a player $i(t)$ reaps a private reward $\pi_t \geq 0$ from winning his own battle, irrespective of his team's success or failure. The value of the private reward π_t is common to the matched players $A(t)$ and $B(t)$. It is determined by the

[11] Throughout the paper, we assume that the two players involved in a particular battle remain unchanged and the matching is complete. The model does not allow teams to strategically and unilaterally set the order in which their players turn up in a sequential contest.

[12] In our setting, the contest would not be terminated unless all $2n + 1$ component battles have been carried out. Konrad and Kovenock (2009) assume that the contest ends immediately once the grand winner is determined. Our main results do not depend on the prevailing termination rule.

characteristics of the particular battlefield and allowed to vary across battlefields. For instance, US Senate membership could have higher value for California politicians than for their Indiana counterparts.

A trivial battle t is no different from a standard static contest in which two players compete for a single prize π_t. It elicits zero effort if $\pi_t = 0$.

B. *Component Battles*

In each component battle t, matched players simultaneously exert their efforts $x_{i(t)}$. Players can be heterogeneous in terms of their innate abilities, and the ability differential is reflected in the possibly different marginal effort costs. Each player $i(t)$'s effort entry $x_{i(t)}$ incurs a constant marginal cost $c_{i(t)} > 0$, which is independently distributed with a cumulative distribution function $F_{i(t)}(\cdot)$. The distribution of each $c_{i(t)}$ is common knowledge, while the realization of $c_{i(t)}$ is privately observed by player $i(t)$ only. The setting flexibly accommodates various forms of heterogeneity and a range of information structures. First, $c_{i(t)}$ is not required to be distributed identically; the distribution function $F_{i(t)}(\cdot)$ is allowed to differ across all players. In particular, $c_{i(t)}$ can be distributed over different supports $[\underline{c}_{i(t)}, \overline{c}_{i(t)}]$. Second, the distribution could be a singleton, making the marginal cost $c_{i(t)}$ publicly known. We will further elaborate on this when enumerating possible variants of our model in Section IC. The assumption of independently distributed marginal effort cost plays a critical role, as it ensures no systematic information asymmetry between teams. We further discuss this assumption in Section IIIB.

C. *Contest Technologies*

In each battle t, a player $i(t)$ wins with a probability of $p_{i(t)}(x_{A(t)}, x_{B(t)})$, with $p_{A(t)}(x_{A(t)}, x_{B(t)}) + p_{B(t)}(x_{A(t)}, x_{B(t)}) = 1$. We do not specify a particular functional form for $p_{i(t)}(x_{A(t)}, x_{B(t)})$, but only assume that the probability is homogeneous of degree zero in players' efforts.

CONDITION 1: $\forall x_{A(t)}, x_{B(t)} \geq 0, \theta > 0$ *and* t, $p_{i(t)}(\theta x_{A(t)}, \theta x_{B(t)}) = p_{i(t)}(x_{A(t)}, x_{B(t)})$.

In addition, our analysis requires that an eligible contest technology induce a unique bidding equilibrium in any two-player one-shot contest with a fixed prize. We now enumerate several models that satisfy Condition 1 and equilibrium uniqueness.

Model 1 (All-Pay Auction with Two-Sided Continuous Incomplete Information): Players' marginal effort costs $c_{i(t)}$ are distributed on nondegenerate intervals, with $\overline{c}_{i(t)} > \underline{c}_{i(t)} > 0$, and have continuously differentiable density functions $f_{i(t)}(\cdot) > 0$. In the battle, the higher bidder always wins. Amann and Leininger (1996) establish a unique pure-strategy Bayesian Nash equilibrium in an equivalent two-player all-pay auction.

Model 2 (Generalized Tullock Contest with Two-Sided Continuous Incomplete Information): Players' marginal costs $c_{i(t)}$ are identically distributed on a nondegenerate interval $[\underline{c}_t, \overline{c}_t]$, and have continuously differentiable density functions $f_t(\cdot) > 0$. A player wins his battle with probability $p_{i(t)}\big(x_{A(t)}, x_{B(t)}\big) = x_{i(t)}^{r_t} / \big[x_{A(t)}^{r_t} + x_{B(t)}^{r_t}\big]$, $i = A, B$, where $r_t \in (0, 1]$. Ryvkin (2010) establishes the existence of a symmetric pure-strategy equilibrium in this model, and Ewerhart (2014) further establishes its uniqueness.[13]

Model 3 (**All-Pay Auction with Discretely Distributed Marginal Costs and Two-Sided Incomplete Information**)**:** Players' marginal costs $c_{i(t)}, \forall i, t$ take a finite number of discrete values and are privately known. A higher bidder always wins. This model is covered by Siegel (2014), who establishes the unique mixed-strategy equilibrium in a more general framework.

Model 4 (**Generalized Tullock Contest with Discretely Distributed Marginal Costs and Two-Sided Incomplete Information**)**:** Cost and information structures are the same as those of Model 3. A generalized Tullock contest, as in Model 2, is adopted to model the winner-selection mechanism. This model is covered by Ewerhart and Quartieri (2013), who establish the unique pure-strategy equilibrium in a more general framework.[14]

Model 5 (**All-Pay Auction with One-Sided Continuous Incomplete Information**)**:** One player's marginal cost is public, while the other's is private and remains continuously distributed. The higher bidder always wins. Morath and Münster (2013) characterize the closed-form solution to the unique equilibrium.[15]

Model 6 (**Generalized Tullock Contest with Complete Information**)**:** Both players' costs are public. Each player wins with a probability $p_{i(t)}\big(x_{A(t)}, x_{B(t)}\big) = x_{i(t)}^{r_t} / \big[x_{A(t)}^{r_t} + x_{B(t)}^{r_t}\big]$, $i = A, B$, where $r_t \in (0, +\infty)$. Szidarovszky and Okuguchi (1997) and Cornes and Hartley (2005) show that when $r_t \in (0, 1]$, there is a unique pure-strategy equilibrium.[16]

Model 7 (**All-Pay Auction with Complete Information**)**:** Model 6 converges into an all-pay auction when r_t approaches infinity. Hillman and Riley (1989) and Baye, Kovenock, and de Vries (1996) verify the existence of a unique equilibrium.

As will be shown later, Condition 1 ensures that contenders' equilibrium bids are linear in the prize. This property plays a critical role. It defines the scope of applicability for our analysis and clearly delineates the boundary of our analysis. The implications will be revealed when our analysis unfolds in Section IIB and be further discussed in Section IIIB.

[13] To our knowledge, the existence of equilibrium remains an open question in this model when $r_t > 1$.

[14] Hurley and Shogren (1998) and Malueg and Yates (2004) studied equilibria in generalized Tullock contests with similar information structures with two or three discrete signals.

[15] Seel (2014) considers a similar setting (Section 4 of his paper) in which only one player has private information.

[16] Malueg and Yates (2006) also established sufficient conditions for the existence and uniqueness of pure-strategy equilibrium. Alcalde and Dahm (2010) and Wang (2010) fully characterize equilibrium bidding strategies for the whole range of $r_t > 0$.

II. Analysis

We first present a simple example to illustrate the distinct features of our pairwise-matched team contests and sketch the key logic that underlies the sharp contrast between the team contests and their counterpart contests of individual players. We then conduct a complete analysis of the general game.

A. *An Example: Symmetric Best-of-Three Contest*

We begin with a benchmark best-of-three contest between two symmetric individual players. We then consider its counterpart contest between two teams, with each team consisting of three players. Assuming $r_t \in (0, 1]$, we adopt Model 6 of Section IC (Generalized Tullock contests with complete information) for each component battle. In both settings, a player's effort incurs a unity marginal cost, which is commonly known. The prize for winning the whole contest is normalized to 1. For simplicity, we assume that battle prizes are zero.

Benchmark: Contest between Individuals.—Two individual players, indexed by $i = A, B$, compete for a prize of value 1. They confront each other in three successive battles, which are indexed by $t = 1, 2, 3$.

The game can be analyzed by backward induction. Suppose that one player wins the first two battles. Battle 3 becomes irrelevant. Next, suppose that each has won one battle. The outcome of battle 3 determines the ultimate winner. In a symmetric equilibrium, each wins with a probability of $1/2$. Let $E(x_3)(> 0)$ be one's expected effort in the equilibrium. Each player thus expects a payoff of $v_3 = 1/2 - E(x_3)$ for participating in this battle.

We then consider players' incentives in battle 2. Assume, without loss of generality, that player 1 is in the lead. He can end the contest and collect a prize of value 1 if he wins battle 2; losing it would force him to participate in the deciding battle, from which he expects a payoff of v_3. Hence, player 1 responds to an "effective prize spread" of $1 - v_3 = 1/2 + E(x_3)$ in battle 2. In contrast, player 2 has an effective prize spread of $v_3 - 0 = 1/2 - E(x_3)$.

A usual strategic-momentum effect or discouragement effect arises, as player 1 values the battle more and therefore is more likely to win. A laggard has to continue to sink costly effort into the third battle if he wins the current one, which dissipates his future rent, thereby attenuating his incentive to remain in the contest. In contrast, the front runner would be forced to endure another costly battle if he loses, which burns future rent and aggravates the pain of losing the current battle. In summary, the laggard has less to win, while the front runner has more to lose. Asymmetry thus arises ex post. Such ex post asymmetry, however, would not appear in a team contest.

Team Contest.—Suppose that one team has won the first two battles; battle 3 becomes irrelevant. If the two teams score evenly in the first two, battle 3 is a symmetric match, and each player wins with a probability of $1/2$.

Consider the immediately preceding battle. Assume that player $A(1)$ has won battle 1 on behalf of team A. If player $A(2)$ wins the battle, the contest ends, and he receives an immediate reward of 1. If he loses, the contest proceeds to battle 3. Player $A(2)$ expects a payoff of $1/2$ from the latter event, because his teammate—i.e., player $A(3)$—may win that battle and secure the prize for their team with probability $1/2$. Hence, player $A(2)$'s effective prize spread is $1 - 1/2 = 1/2$. If player $B(2)$ wins the battle, the contest proceeds to the deciding battle. Player $B(2)$ would stand a chance of winning the contest with probability $1/2$, as player $B(3)$ may win battle 3. He thus expects a payoff of $1/2$ from winning his battle. If he loses, the contest ends. Hence, player $B(2)$'s effective prize spread is $1/2 - 0 = 1/2$.

The battle remains symmetric, and the strategic-momentum or discouragement effect does not arise. Each player performs in only one battle. Because of this fact, player $A(2)$ on the leading team suffers less than his counterpart in the benchmark contest between individuals, because a prolonged contest does not cost his additional effort; player $B(2)$ on the lagging team values his own win more than his counterpart in the contest between individuals, because the deciding battle, from his viewpoint, is an effortless fair draw.

In the next subsection, we provide a complete analysis in the general setup delineated in Section I. The fundamentals we observe in this stylized example remain valid.

B. *General Analysis*

Let $v_i(k_A, k_B)$ denote the *continuation value* to team i when the contest is in a state (k_A, k_B), which depicts the gross payoff players on team i expect for winning the team prize when it is assessed in the given state. As the value of the prize is normalized to 1, the *continuation value* coincides with the team's conditional winning probability. By definition, we must have $v_i(k_A, k_B) = 1$ if $k_i \geq n + 1$, and $v_i(k_A, k_B) = 0$ if $k_j \geq n + 1$: in the former case, team i has won, and all its players receive a prize of value 1; in the latter, the team has lost.

Consider an arbitrary battle t in a state (k_A, k_B), with $t = k_A + k_B + 1$. For player $A(t)$, the contest reaches a state $(k_A + 1, k_B)$ if he wins, and the continuation value for team A's players becomes $v_A(k_A + 1, k_B)$; the contest reaches a state $(k_A, k_B + 1)$ if he loses, and the continuation value correspondingly becomes $v_A(k_A, k_B + 1)$. As illustrated by the best-of-three example, each player in the team contest turns up only once and does not bear the cost required by future battles. Besides the private reward π_t, a player is only concerned about how the outcome of the current battle affects his prospect for winning the team prize. Hence, the effective prize spreads for players $A(t)$ and $B(t)$ in this battle are given, respectively, by $\pi_t + [v_A(k_A + 1, k_B) - v_A(k_A, k_B + 1)]$ and $\pi_t + [v_B(k_A, k_B + 1) - v_B(k_A + 1, k_B)]$.

We now present two key observations about the game, which pave the way for all our main analyses.

OBSERVATION 1 (Common-Value Battles): *In each battle t, players $A(t)$ and $B(t)$ have a common effective prize spread of $V_t(k_A, k_B) = \pi_t + \Delta v(k_A, k_B)$, where $\Delta v(k_A, k_B) = v_A(k_A + 1, k_B) - v_A(k_A, k_B + 1) = v_B(k_A, k_B + 1) - v_B(k_A + 1, k_B)$.*

Observation 1 says that matched players always equally value winning their battle, regardless of the state of the contest. The common prize spread of $V_t(k_A, k_B)$

directly stems from a simple fact: the sum of teams' continuation values always amounts to one, i.e., $v_A(k_A, k_B) + v_B(k_A, k_B) \equiv 1$, since the team prize is to be won by one and only one team.

By Observation 1, a fixed pair of matched players must be equally motivated, although the particular size of the common valuation depends on the specific state (k_A, k_B). The following observation follows from the assumed homogeneity-of-degree-zero contest technology (Condition 1).[17]

OBSERVATION 2 (Homogeneity-of-Degree-One Equilibrium Bidding Strategies): *Consider an arbitrary battle t that satisfies Condition 1 and has a common prize spread $V_t > 0$. Suppose that a unique equilibrium exists, which could be in either pure or mixed strategies. Then a player $i(t)$ with cost $c_{i(t)}$ exerts an equilibrium effort $b_{i(t)}\left(c_{i(t)}; V_t\right) = V_t \xi_{i(t)}\left(c_{i(t)}\right)$,[18] where $\xi_{i(t)}(\cdot)$, $i = A, B$ is his equilibrium bidding strategy when $V_t = 1$, which is solely determined by the matched players' cost distributions $\left(F_{A(t)}(\cdot), F_{B(t)}(\cdot)\right)$.*

Observation 2 states that in the unique bidding equilibrium, whenever it exists, players' equilibrium bids are linear in the (positive) common valuation V_t: a change in V_t scales their equilibrium efforts up or down by the same factor. With these preliminaries, we are ready to present our main results.

History Independence and Prize Spreads.—Observation 2 shows that in each battle, players' winning probabilities are independent of their common valuation, provided that the common valuation remains positive. Denote by $\mu_{i(t)}$ the expected probability of a player $i(t)$'s winning his battle t when $V_t > 0$. We call $\left(\mu_{A(t)}, \mu_{B(t)}\right)$ the *stochastic winning outcome* of a battle t, which, by Observation 2, is solely determined by players' marginal effort cost distributions $(F_{A(t)}(\cdot), F_{B(t)}(\cdot))$. Observations 1–2, together, imply the following.

THEOREM 1 (History Independence of Winning Outcome): *Consider an arbitrary battle t in a state (k_A, k_B) with $t = k_A + k_B + 1$. Whenever $V_t(k_A, k_B) > 0$, the stochastic outcome $(\mu_{A(t)}, \mu_{B(t)})$ of the battle is determined solely by the matched players' effort cost distributions $(F_{A(t)}(\cdot), F_{B(t)}(\cdot))$, and thus is independent of (k_A, k_B).*

To elaborate on our result's implication, we begin with a simple case in which all battles have positive private prizes, i.e., $\pi_t > 0, \forall t \in \{1, \dots, 2n+1\}$. The contest becomes a series of independent lotteries with fixed odds $\left(\mu_{A(t)}, \mu_{B(t)}\right)$. Fix players' matching, and two observations are immediate. First, each team's likelihood of winning the contest remains constant even if battles are reordered. Second, note that Observations 1–2, which underpin Theorem 1, are not artifacts of the sequential setting. The implications of Theorem 1 would continue to hold even if (some) battles

[17] This observation is due to the following result: for any effort profile $x_{A(t)}$, $x_{B(t)}$ and $V_t(> 0)$, player i's expected payoff is $V_t p_{i(t)}\left(x_{A(t)}, x_{B(t)}\right) - c_{i(t)} x_{i(t)} = V_t\left[p_{i(t)}\left(\frac{x_{A(t)}}{V_t}, \frac{x_{B(t)}}{V_t}\right) - c_{i(t)}\frac{x_{i(t)}}{V_t}\right]$ when his type is $c_{i(t)}$. A detailed proof is available from the authors upon request.

[18] We allow $\xi_{i(t)}\left(c_{i(t)}\right)$ to be a random variable to accommodate mixed-strategy equilibrium.

took place simultaneously. The teams' ex ante winning likelihoods, accordingly, would not vary either.

Further, consider players' contingent prize spread $V_t(k_A, k_B) = \pi_t + \Delta v(k_A, k_B)$ in a state (k_A, k_B). The second term, $\Delta v(k_A, k_B)$, in $V_t(k_A, k_B)$ depicts the incentive provided by the team prize. Note that a player $i(t)$, by winning this battle, derives additional benefit from the team prize if and only if the battle turns out to be the tie-breaker. The incentive provided by the team prize, accordingly, must be discounted by the probability of the event that battle t is ex post pivotal. Define the function $\theta_i(h\,|\,m)|_t^{t+m-1}$, with $t \in \{1, 2, \ldots, 2n+1\}$, $m \in \{0, 1, 2, \ldots, (2n+1)-(t-1)\}$ and $h \in \{0, 1, \ldots, m\}$, to be the probability that players $i(t)$ to $i(t+m-1)$ will win exactly h of the m battles (battles t to $t+m-1$), provided that the stochastic winning outcome $(\mu_{A(t)}, \mu_{B(t)})$ applies in all the m battles.[19] Given that battles are all independent lotteries in this case, the probability of the battle's being pivotal is simply $\theta_i(n-k_i\,|\,2n-k_A-k_B)|_{t+1}^{2n+1}$, $i = A, B$.[20]

The same logic allows us to evaluate ex ante the incentive provided by the team prize to a player $i(t)$, which aggregates all possible states (k_A, k_B) in which his battle could take place. It must be given by the ex ante probability of battle t's being pivotal to the contest. Define $\theta(n\,|\,2n)|_{-t} \equiv \theta_i(n\,|\,2n)|_{-t}$, $\forall i \in \{A, B\}$, which is the probability that each team will win exactly n out of the $2n$ battles other than battle t, assuming that the stochastic winning outcome $(\mu_{A(t')}, \mu_{B(t')})$, $\forall t' \in \{1, 2, \ldots, 2n+1\}$ always applies.[21] In this simple case, the ex ante probability of battle t's being pivotal is simply $\theta(n\,|\,2n)|_{-t}$.

Hence, the ex ante expected common prize spread for players $i(t)$ must be $\pi_t + \theta(n\,|\,2n)|_{-t}$, which is determined solely by the other players' marginal cost distributions. Under fixed matching, it would not vary if battles are ordered differently, or (some) battles are held simultaneously. By Observation 2, the ex ante expected effort contributed by a fixed pair of players would also be constant.

The fixed stochastic winning outcome $(\mu_{A(t)}, \mu_{B(t)})$ in Theorem 1, however, does not apply universally, and the independence does require $V_t > 0$. It could lose its bite if $\pi_t = 0$: The battle could have become trivial when taking place, in which case it elicits zero effort and its winning outcome is determined by the default tie-breaking rule. Our analysis will demonstrate that all of the above observations would hold even if π_t were allowed to be zero.

LEMMA 1: *Consider an arbitrary battle t.*

(*i*) (*Contingent expected prize spreads*) *Suppose that battle t takes place in a state* (k_A, k_B) *with* $t = k_A + k_B + 1$. *Players* $A(t)$ *and* $B(t)$ *have a common contingent prize spread* $V_t(k_A, k_B) = \pi_t + \theta_i(n-k_i|\,2n-k_A-k_B)|_{t+1}^{2n+1}$, $i = A, B$.

[19] We define $\theta_i(0|0)|_t^{t-1} = 1$, $\theta_i(h|m)|_t^{t+m-1} = 0$ for $h < 0$ and $m \geq 0$, and $\theta_i(h|m)|_t^{t+m-1} = 0$ for $h > m \geq 0$.

[20] Note that $\theta_A(n-k_A|2n-k_A-k_B)|_{t+1}^{2n+1} \equiv \theta_B(n-k_B|2n-k_A-k_B)|_{t+1}^{2n+1}$, because a team i's winning $n-k_i$ battles is equivalent to its rival team j's winning $n-k_j$ of them.

[21] Note that we must have $\theta_A(n|2n)|_{-t} = \theta_B(n|2n)|_{-t}$: when a team wins n out of the $2n$ battles, the rival team must prevail in exactly n other battles as well.

(*ii*) (*Ex ante expected prize spreads*) *The ex ante expected prize spread* $E_{(k_A, k_B)}[V_t(k_A, k_B)]$ *of battle t is* $\pi_t + \theta(n\,|\,2n)|_{-t}$.

Lemma 1 is not straightforward in a dynamic setting when allowing for $\pi_t = 0$, since the stochastic winning outcome $(\mu_{A(t)}, \mu_{B(t)})$ would not apply when battle t turns out to be trivial. The lemma nevertheless shows that the impact of an early battle on future battles appears to fade away, as if all battles took place concurrently as independent lotteries. The puzzle can be resolved by the following argument. Consider a nontrivial battle t in a given state (k_A, k_B) with $t = k_A + k_B + 1$. Team A's conditional winning odds, or its continuation value, can be written as

$$(1) \qquad v_A(k_A, k_B) = \sum_{m=k_B+n+1}^{2n+1} \left[\mu_{A(m)} \cdot \theta_A(n - k_A | m - t)\Big|_t^{(m-1)}\right].$$

To win the contest, the team could secure its $(n + 1)$th win in any battle $m \geq k_B + n + 1$. The expression (1) adds up all these possibilities: Each item $\mu_{A(m)} \cdot \theta_A(n - k_A|m - t)_t^{(m-1)}$ is the probability of securing the $(n + 1)$th win in a battle m. In each event, all the battles involved in the calculation are nontrivial and thus have strictly positive prize spreads. The winning outcomes in subsequent trivial battles—i.e., battles $t' > m$—are irrelevant. The dynamic linkage thus dissolves.

Sequence Independence.—We now formally evaluate the ramifications of alternative sequencing arrangements on the various properties of the contest, including teams' ex ante winning odds, players' effort supply, and their ex ante expected payoffs. Fix the pairwise matching between players on the two teams. Let each pair be assigned to a fixed battlefield and compete for a fixed private prize. We reshuffle the sequence of these (heterogeneous) battles.

The effects of such reshuffling on players' behavior and the outcome of the contest remain unclear. Suppose, for instance, that one team possesses a lopsided advantage over the other in one battle. This battle would certainly accumulate an advance if it were scheduled for an early slot, while it could appear to be wasted if it were ordered in a late slot and took place as a trivial one. It thus remains unclear how teams' ex ante winning likelihoods would be affected. Calculating the probability is straightforward if all battles award private prizes, in which case the contest reduces to a series of independent lotteries with fixed winning odds. The aforementioned dynamic linkage, however, remains a concern when private prizes can be zero. The prevailing sequencing arrangement would affect the probability of a battle's being trivial and, in turn, its stochastic outcome. For instance, a battle ordered at an early position would never be trivial. It would become possible, however, if the battle were scheduled for a late slot. Therefore, if a battle does not award a private prize, its stochastic outcome is not immune to reshuffling.

Index by $g \in \{1, 2, \ldots, 2n + 1\}$ the (fixed) pairs of matched players, and denote by $i(g)$ the player from team i in pair g. We use $t(g)$ to denote the (variable) temporal order of the battle assigned to pair g in an arbitrary fixed sequence of battles. The following is obtained.

THEOREM 2 (Sequence Independence):

(*i*) *Neither the ex ante expected prize spread of a battle between an arbitrary pair g, nor its expected total effort, nor the ex ante expected total effort of the entire contest, depends on the battle sequence.*

(*ii*) *Teams' ex ante expected likelihoods are independent of the battle sequence.*

(*iii*) *Players' ex ante expected payoffs are independent of the battle sequence.*

Theorem 2(i) is immediately implied by Lemma 1(ii) and Observation 2. The following argument explains Theorem 2(ii) in the general case. Under a given sequence, a team's ex ante expected winning odds are given by

$$v_i(0,\,0) \;=\; \sum_{m=n+1}^{2n+1} \left[\mu_{i(m)} \cdot \theta_i(n|m-1)\Big|_1^{(m-1)}\right],\; i \;=\; A,\, B.$$

A team could secure its $(n+1)$th win in any battle m, with $m \geq n+1$, and the expression aggregates all these possibilities. Each of these events occurs with a probability $\mu_{i(m)} \cdot \theta_i(n|m-1)_1^{(m-1)}$, which involves no trivial battle. Therefore, $v_i(0,0)$ does not depend on how a trivial battle is resolved. Evaluating $v_i(0,0)$ in a general setting that allows for zero battle prizes is no different from doing it in the previously discussed limited case: in that case, battles are always resolved by fixed winning odds, and a team's winning odds are simply the probability of its players being drawn as winners in at least $n+1$ independent lotteries; reshuffling does not vary it.

Theorem 2(iii) thus follows. A player $i(g)$'s ex ante expected payoff includes: (i) the discounted payoff from the team prize, i.e., his team's expected winning odds; (ii) the expected payoff from winning the private reward, i.e., $\mu_{i(g)}\pi_g$, where π_g denotes the battle prize for pair g; and (iii) his ex ante expected effort costs $\left[\pi_g + \theta(n|2n)\big|_{-t(g)}\right] \cdot E_{c_{i(g)}}\left[c_{i(g)} E\xi_{i(g)}(c_{i(g)})\right]$. None of these depend on the prevailing sequencing arrangement.

Temporal-Structure Independence.—Lemma 1 and Theorems 1 and 2, as well as the logic that underlies these results, break the dynamic linkage among battles in a sequential setting and pave the way for a broader analysis that accommodates a full spectrum of temporal structures.

Component battles can be carried out (partially) simultaneously with full flexibility. More specifically, the $2n+1$ component battles are partitioned into $Z \leq 2n+1$ clusters. The battles included in the same cluster are carried out simultaneously, while different clusters are carried out sequentially. Players in a battle do not observe the outcomes of parallel battles within the same cluster, while they learn the outcomes of battles within previous clusters. This setup accommodates all possible temporal structures. With $Z = 2n+1$, the sequential setting considered in the baseline setting is restored. With $Z = 1$, a completely simultaneous setting obtains, in which all battles are carried out at the same time. The analysis allows us to explore the ramifications of a contest's temporal structure.

Theorem 1 (history independence) continues to hold, as Observations 1–2 remain valid. The following theorem depicts the game's general neutrality to temporal structure. The formal proof is relegated to the online Appendix.

THEOREM 3 (Temporal-Structure Independence):

(*i*) *Neither the ex ante expected prize spread in a battle between an arbitrary pair of matched players, nor the ex ante expected effort of the battle, nor that of the whole contest depends on the temporal structure.*

(*ii*) *Teams' ex ante winning probabilities are independent of the temporal structure.*

(*iii*) *Players' ex ante expected payoffs are independent of the temporal structure.*

Theorem 3 provides a unified account of team contests with pairwise-matched battles under any temporal structure. All three parts of Theorem 3 hold by similar arguments laid out in previous subsections. Prize spreads and teams' ex ante expected winning odds, for instance, can be obtained—as discussed in Section IIB—by simply assuming that battles are independent lotteries with fixed winning odds.

In summary, the ex ante expected winning outcome, the ex ante expected effort of the contest, and players' ex ante expected payoffs are entirely independent of the prevailing temporal structure of the contest, regardless of how the battles are clustered.

III. Implications, Extensions, and Caveats

A. *Implications*

Our analysis provides a thorough account of the equilibrium behavior and outcomes in contests between teams. Our neutrality results, which break the dynamic linkage among battles, sharply contrast with the conventional wisdom in the literature.

These results shed light on contest design in various contexts. For instance, we have addressed a classical issue on intermediate feedback policy in contests: does it pay to provide intermediate feedback to contestants in a dynamic contest?[22] A (partially) simultaneous temporal structure in our setting can be equivalently interpreted as a contest with limited intermediate feedback, which prevents players from perfectly observing past plays. Our analysis in Section IIB thus implies that in our context, the feedback or disclosure policy affects neither the performance nor the outcome of the contest.

Our analysis also provides a novel perspective on the design of electoral systems. For instance, how does the temporal format of an electoral competition affect its

[22] The literature on communication and interim performance feedback in dynamic contests includes Gershkov and Perry (2009); Aoyagi (2010); Ederer (2010); Gürtler and Harbring (2010); and Goltsman and Mukherjee (2011). In contrast to our setting, players in these studies compete on a single task, and the winner is determined by players' accumulated output in carrying out the task.

outcome and the resulting campaign expenditures? These issues have been investigated in the context of electoral competitions between individual candidates, but have yet to be investigated in competitions between political parties or alliances. As discussed in the introduction, different temporal formats are observed in practice. Our analysis provides a theoretical benchmark to identify the distinctive strategic elements embedded in such scenarios.

Furthermore, in many sports tournaments with team titles, e.g., the Thomas Cup (men's badminton), athletes are typically sorted and matched by their professional rankings. The fixed matching of players between two teams leaves open the question of how to order these matches. For instance, should the match between top-ranked players be scheduled for an earlier or a later round? Our sequence-independence result sheds light on these issues.

Two remarks are in order to stress the subtle and important roles played by the distinct competition structure of team contests with pairwise battles. First, the unique form of team production considered in our analysis largely contributes to the results. Consider a hypothetical situation in which two soccer teams meet each other repetitively. In that context, players on a team simply join forces in each match, and every player has to participate in a series of battles. The usual discouragement would still loom large, as one is required to supply his effort repetitively on the team. In contrast, in our setup, each player stands alone on behalf of his team in only one battle, which eliminates the intertemporal cost trade-offs. Second, the same fact renders our study complementary to that of Konrad and Kovenock (2010). They consider a situation in which two players meet repetitively, but their effort costs are redrawn in every battle. They find that the uncertainty regarding costs mitigates dynamic discouragement, but does not entirely eliminate it, because a player is still concerned about his future cost.

B. *Extensions and Caveats*

Our analysis is conducted in a relatively stylized setting. The robustness and caveats of our results should be examined carefully to identify their limits and scope for proper application.

Our results do not lose their bite even when one team is allowed to have a head start—e.g., a party has more seats in the Senate before an election due to its past success or turnover—such that it could secure the ultimate victory by winning a smaller number of battles than its opponent. Put simply, a head start is equivalent to an early lead in the symmetric contest analyzed in our baseline setting, which, by Theorem 1, does not distort future competitions. All of the primary predictions can naturally be retained.

A few caveats can be immediately inferred from our analysis. For instance, our neutrality results depend crucially on Observation 2—i.e., the linear relation between matched players' equilibrium bids and their common prize spreads—which ultimately requires a homogeneity-of-degree-zero contest technology (Condition 1). Absent this property, the stochastic outcome would in general depend on the particular size of the prize spread, which is determined by the specific state of the contest, although matched players still equally value winning the battle. The stochastic outcome would no longer be immune to changes in the state of the contest unless players are symmetric, in which case matched players always win their battle with equal probability. By this logic, one can expect similar implications (i) when players' effort cost function is nonlinear, or (ii) when players are not risk neutral.

Several extensions will expand our understanding of the robustness of our main results and allow us to identify the factors that would nullify the current predictions.

Uneven Winning Scores and Unequally Weighted Battles.—A direct extension is to allow component battles to carry different weights. Specifically, consider a contest with $T \geq 2$ successive component battles. A team i is awarded a score s_t^i if its player $i(t)$ wins the battle t. Each team can maximally score 1 if its players prevail in all battles, i.e., $\sum_{t=1}^{T} s_t^i = 1, \forall i$. A team wins if and only if it obtains a higher accumulated score $\tilde{s}^i$. The score awarded for winning a battle may vary across battles for a given team, i.e., $s_t^i \neq s_{t'}^i$ for $t \neq t'$. It may also vary across teams for a given battle, i.e., $s_t^A \neq s_t^B$. Our baseline model becomes a special case of this extended setting.

Let the state of the contest be summarized by a tuple $(\tilde{s}^A, \tilde{s}^B)$, which indicates each team's accumulated score. Consider an arbitrary battle t in a state $(\tilde{s}^A, \tilde{s}^B)$. Again, let $v_i(\tilde{s}^A, \tilde{s}^B)$ be a team i's continuation value. Players' prize spreads are given by $V_{A(t)} = \pi_t + \left[v_A\left(\tilde{s}^A + s_t^A, \tilde{s}^B\right) - v_A\left(\tilde{s}^A, \tilde{s}^B + s_t^B\right)\right]$ and $V_{B(t)} = \pi_t + \left[v_B\left(\tilde{s}^A, \tilde{s}^B + s_t^B\right) - v_B\left(\tilde{s}^A + s_t^A, \tilde{s}^B\right)\right]$, respectively.

A simple fact still holds: teams' expected winning odds always sum up to 1. Observation 1 thus remains intact, i.e., $V_{A(t)} = V_{B(t)}$. Players of battle t always equally value winning this battle, which retains Observation 2. These two key observations lead to the same qualitative predictions as those in our baseline setting.

Asymmetry in Valuations.—Our analysis has assumed a symmetric prize structure, i.e., both the team prize and battle prizes are commonly valued by matched players. Absent this assumption, our model's predictions would persist if and only if players receive no private reward from winning their own battles.

Let each player $i(t)$ have a valuation of $\tau_{i(t)} > 0$ for the team prize. Recall that $v_i(k_A, k_B)$ is the probability of a team i's winning the contest assessed in a state (k_A, k_B). For a battle t, the two matched players' effective prize spreads in the current state are given, respectively, by $V_{A(t)}(k_A, k_B) = \pi_t + [v_A(k_A + 1, k_B) - v_A(k_A, k_B + 1)]\tau_{A(t)}$ and $V_{B(t)}(k_A, k_B) = \pi_t + [v_B(k_A, k_B + 1) - v_B(k_A + 1, k_B)] \times \tau_{B(t)}$.

Recall that $\Delta v(k_A, k_B) = v_A(k_A + 1, k_B) - v_A(k_A, k_B + 1) = v_B(k_A, k_B + 1) - v_B(k_A + 1, k_B)$ by Observation 1. When a battle t involves no private reward, i.e., $\pi_t = 0$, we must have $V_{i(t)}(k_A, k_B) = \Delta v(k_A, k_B)\tau_{i(t)}$, $i = A, B$. The state of the contest only affects the common factor of $\Delta v(k_A, k_B)$ in players' effective prize spreads. The ratio between players' prize spreads must remain constant regardless of the prevailing state (k_A, k_B). With a contest technology that satisfies Condition 1, one can verify that when $V_{i(t)}$ are scaled up or down by a common factor $\gamma(> 0)$, their equilibrium bids would be scaled up or down by the same factor γ.[23] Players'

[23] This result can be obtained similarly as Observation 2. A formal proof is available from the authors upon request.

winning odds are thus determined solely by their initial valuations of the team prize $\tau_{i(t)}$ and their effort cost distributions.

The same, however, does not hold if positive private rewards exist. One's private reward is unaffected by the state of the contest. As a result, matched players' overall prize spreads would vary by different proportions when the state of the contest changes; a discouragement effect thus emerges. This effect is partially muted, however, because players' expected payoff from the team prize is still discounted "symmetrically."

Information Asymmetry between Teams.—In Sections IB and IC, we have demonstrated that our model accommodates a wide span of information structures for players' marginal effort costs. However, additional complications arise when information asymmetry exists between teams. By the assumption of independently distributed marginal costs, we have implicitly assumed that a player's cost characteristics are symmetrically known to all other players, i.e., both his own teammates and players on the rival team. If, instead, players possess private information about their teammates, their strategic trade-offs and behavior would change dramatically. For instance, a player might give up prematurely in response to unfavorable early outcomes if he knows that his (weak) teammates are unlikely to win future battles, while his opponent does not. Such private information leads matched players' assessments of their teams' winning odds to diverge. Observations 1–2 would no longer hold, which nullifies our neutrality results. Inter-team information asymmetry is not uncommon. For instance, athletes on a team often attend training camp together before major tournaments, which affects all participants' competence and/or allows them to learn about others' skill levels.

IV. Concluding Remarks

This paper examines a pairwise-matched multi-battle contest between two teams. We provide a unified framework to analyze such contests under a full spectrum of temporal structures. We establish that the stochastic outcomes of subsequent battles can be independent of the current state of the contest and identify the conditions under which the neutrality arises. Under these conditions, teams' expected winning odds and the expected total effort of the contest are independent of the sequencing arrangement and the temporal structure of the contest.

Our paper contributes to the burgeoning literature on multi-battle contests by allowing for collective action. The analysis illustrates the fundamental difference in players' strategic mindsets between contests of individuals and pairwise-matched team contests.

The main logic of our baseline analysis remains robust for many variations of the basic model. However, we have identified several caveats that could nullify the current predictions, which deserve to be explored more formally in future studies. Our results, therefore, should be interpreted with caution when being applied to specific contexts.

Appendix

PROOF OF LEMMA 1:

We first consider Lemma 1(i). We first show the contingent prize spread. We need to show that

$$\Delta v(k_A, k_B) = v_A(k_A + 1, k_B) - v_A(k_A, k_B + 1)$$
$$= \theta_A(n - k_A|2n - k_A - k_B)\Big|_{t+1}^{2n+1},$$

where $t = k_A + k_B + 1$. When $k_A = k_B = n$, clearly we have $\Delta v(k_A, k_B) = 1$ that equals $\theta_A(n - k_A|2n - k_A - k_B)\Big|_{t+1}^{2n+1} = 1$; when $\max\{k_A, k_B\} \geq n + 1$, clearly we have $\Delta v(k_A, k_B) = 0$ that equals $\theta_A(n - k_A|2n - k_A - k_B)\Big|_{t+1}^{2n+1} = 0$. We now focus on the case where $k_A, k_B \leq n$ and $\min\{k_A, k_B\} < n$. Note that in this case, battle $k_A + k_B + 1$ is nontrivial. Thus we have $v_A(k_A, k_B) = \mu_{A(k_A+k_B+1)} v_A(k_A + 1, k_B) + \big(1 - \mu_{A(k_A+k_B+1)}\big) v_A(k_A, k_B + 1)$ by definition. One can then verify that $\Delta v(k_A, k_B) = \mu_{A(k_A+k_B+1)} \Delta v(k_A + 1, k_B) + \big(1 - \mu_{A(k_A+k_B+1)}\big)\Delta v(k_A, k_B + 1)$. Applying this result, it is clear that $\Delta v(k_A, k_B) = \theta_A(n - k_A|2n - k_A - k_B)\Big|_{t+1}^{2n+1}$ for all $k_A, k_B \leq n$ and $\min\{k_A, k_B\} < n$ can be verified iteratively by mathematical induction.

We now turn to the ex ante expected prize spread in Lemma 1(ii). Let $\Pr((k_A, k_B)|t)$ denote the ex ante expected probability that battle t will take place in a state (k_A, k_B). Note

$$E(V_t) = \sum_{k_A+k_B=t-1} \Pr((k_A, k_B)|t) V_t(k_A, k_B),$$

with $\sum_{k_A+k_B=t-1} \Pr((k_A, k_B)|t) \equiv 1$ regardless of t.

We first look at the case where $t \leq n + 1$. In this case, we must have $\max\{k_A, k_B\} \leq n$ and $\min\{k_A, k_B\} < n$. By Theorem 1, we have

$$E(V_t) = \sum_{k_A+k_B=t-1} \Pr((k_A, k_B)|t) V_t(k_A, k_B)$$
$$= \pi_t + \sum_{k_A+k_B=t-1} \theta_A(k_A|k_A + k_B)\Big|_1^{t-1} \times \theta_A(n - k_A|2n - k_A - k_B)\Big|_{t+1}^{2n+1}$$
$$= \pi_t + \theta_A(n|2n)\Big|_{-t} = \pi_t + \theta_B(n|2n)\Big|_{-t} = \pi_t + \theta(n|2n)\Big|_{-t},$$

where $\theta_i(n|2n)\Big|_{-t}$ is the overall probability of team i's winning exactly n out of the $2n$ battles (nontrivial) other than battle t.

We now turn to the case where $t \geq n + 2$. This case can be divided into the following subcases:

(*i*) When $\max\{k_A, k_B\} \geq n + 1$, $V_t(k_A, k_B) = \pi_t$;

(*ii*) When $\max\{k_A, k_B\} \leq n - 1$, $V_t(k_A, k_B) = \pi_t + \theta_A \times (n - k_A | 2n - k_A - k_B)\big|_{t+1}^{2n+1}$;

(*iii*) When $k_A = k_B = n$, $V_t(k_A, k_B) = \pi_t + \theta_A(n - k_A | 2n - k_A - k_B)\big|_{t+1}^{2n+1} = \pi_t + 1$;

(*iv*) When $k_A = n, k_B < n$, or $k_A < n, k_B = n$, we have

$$V_t(k_A, k_B) = \pi_t + \theta_A(n - k_A | 2n - k_A - k_B)\big|_{t+1}^{2n+1}.$$

Hence, when $t \geq n + 2$, we must have

$$EV_t = \pi_t + \sum_{\substack{k_A + k_B = t-1 \\ k_A \leq n,\ k_B \leq n}} \theta_A(k_A | k_A + k_B)\big|_1^{t-1} \cdot \theta_A(n - k_A | 2n - k_A - k_B)\big|_{t+1}^{2n+1}$$

$$= \pi_t + \theta_A(n|2n)\big|_{-t} = \pi_t + \theta_B(n|2n)\big|_{-t} = \pi_t + \theta(n|2n)\big|_{-t}. \blacksquare$$

REFERENCES

Alcalde, José, and Matthias Dahm. 2010. "Rent Seeking and Rent Dissipation: A Neutrality Result." *Journal of Public Economics* 94 (1-2): 1–7.

Aldrich, John H. 1980. "A Dynamic Model of Presidential Nomination Campaigns." *American Political Science Review* 74 (3): 651–69.

Amann, Erwin, and Wolfgang Leininger. 1996. "Asymmetric All-Pay Auctions with Incomplete Information: The Two-Player Case." *Games and Economic Behavior* 14 (1): 1–18.

Aoyagi, Masaki. 2010. "Information Feedback in a Dynamic Tournament." *Games and Economic Behavior* 70 (2): 242–60.

Baye, Michael R., Dan Kovenock, and Casper G. de Vries. 1996. "The All-pay Auction with Complete Information." *Economic Theory* 8 (2): 291–305.

Cornes, Richard, and Roger Hartley. 2005. "Asymmetric Contests with General Technologies." *Economic Theory* 26 (4): 923–46.

Cox, Gary W., and Eric Magar. 1999. "How Much is Majority Status in the U.S. Congress Worth?" *American Political Science Review* 93 (2): 299–309.

Ederer, Florian. 2010. "Feedback and Motivation in Dynamic Tournaments." *Journal of Economics and Management Strategy* 19 (3): 733–69.

Ewerhart, Christian. 2014. "Unique Equilibrium in Rent-Seeking Contests with a Continuum of Types." *Economics Letters* 125 (1): 115–18.

Ewerhart, Christian, and Federico Quartieri. 2013. "Unique Equilibrium in Contests with Incomplete Information." University of Zurich Department of Economics Working Paper 133.

Gershkov, Alex, and Motty Perry. 2009. "Tournaments with Midterm Reviews." *Games and Economic Behavior* 66 (1): 162–90.

Goltsman, Maria, and Arijit Mukherjee. 2011. "Interim Performance Feedback in Multistage Tournaments: The Optimality of Partial Disclosure." *Journal of Labor Economics* 29 (2): 229–65.

Gürtler, Oliver, and Christine Harbring. 2010. "Feedback in Tournaments under Commitment Problems: Experimental Evidence." *Journal of Economics and Management Strategy* 19 (3): 771–810.

Harris, Christopher, and John Vickers. 1987. "Racing with Uncertainty." *Review of Economic Studies* 54 (1): 1–21.

Hartog, Chris Den., and Nathan W. Monroe. 2008. "The Value of Majority Status: The Effect of Jeffords's Switch on Asset Prices of Republican and Democratic Firms." *Legislative Studies Quarterly* 33 (1): 63–84.

Hillman, Arye L., and John G. Riley. 1989. "Politically Contestable Rents and Transfers." *Economics & Politics* 1 (1): 17–39.

Hurley, Terrance M., and Jason F. Shogren. 1998. "Asymmetric Information Contests." *European Journal of Political Economy* 14 (4): 645–65.

Klumpp, Tilman, and Mattias K. Polborn. 2006. "Primaries and the New Hampshire Effect." *Journal of Public Economics* 90 (6-7): 1073–1114.

Konrad, Kai A. 2009. *Strategy and Dynamics in Contests*. Oxford: Oxford University Press.

Konrad, Kai A., and Dan Kovenock. 2009. "Multi-Battle Contests." *Games and Economic Behavior* 66 (1): 256–74.

Konrad, Kai A., and Dan Kovenock. 2010. "Contests with Stochastic Abilities." *Economic Inquiry* 48 (1): 89–103.

Kovenock, Dan, and Brian Roberson. 2009. "Is the 50-State Strategy Optimal?" *Journal of Theoretical Politics* 21 (2): 213–36.

Malueg, David A., and Andrew J. Yates. 2004. "Rent Seeking with Private Values." *Public Choice* 119 (1–2): 161–78.

Malueg, David A., and Andrew J. Yates. 2006. "Equilibria in Rent-Seeking Contests with Homogeneous Success Functions." *Economic Theory* 27 (3): 719–27.

Malueg, David A., and Andrew J. Yates. 2010. "Testing Contest Theory: Evidence from Best-of-Three Tennis Matches." *Review of Economics and Statistics* 92 (3): 689–92.

Morath, Florian, and Johannes Münster. 2013. "Information Acquisition in Conflicts." *Economic Theory* 54 (1): 99–129.

Ryvkin, Dmitry. 2010. "Contest with Private Costs: Beyond Two Players." *European Journal of Political Economy* 26 (4): 558–67.

Seel, Christian. 2014. "The Value of Information in Asymmetric All-Pay Auctions." *Games and Economic Behavior* 86: 330–38.

Siegel, Ron. 2014. "Asymmetric All-Pay Auctions with Interdependent Valuations." *Journal of Economic Theory* 153: 684–702.

Snyder, James M. 1989. "Election Goals and the Allocation of Campaign Resources." *Econometrica* 57 (3): 637–60.

Strumpf, Koleman S. 2002. "Strategic Competition in Sequential Election Contests." *Public Choice* 111 (3–4): 377–97.

Szidarovszky, Ferenc, and Koji Okuguchi. 1997. "On the Existence and Uniqueness of Pure Nash Equilibrium in Rent-Seeking Games." *Games and Economic Behavior* 18 (1): 135–40.

Wang, Zhewei. 2010. "The Optimal Accuracy Level in Asymmetric Contests." *B.E. Journal of Theoretical Economics* 10 (1).

This article has been cited by:

1. Qiang Fu, Changxia Ke, Fangfang Tan. 2015. "Success breeds success" or "Pride goes before a fall"?. *Games and Economic Behavior* . [CrossRef]