



Disclosure policy in a multi-prize all-pay auction with stochastic abilities



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HIGHLIGHTS

- We investigate whether a contest organizer should disclose private information about bidders' abilities.
- A multi-prize all-pay auction model is considered.
- We find that concealing the information elicits higher expected total effort.
- We find that the rent-dissipation rate of the contest does not depend on the disclosure policy.

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ABSTRACT

This paper investigates whether a contest organizer should disclose private information about bidders' abilities in a multi-prize all-pay auction. Bidders' abilities are randomly distributed and observed by the contest organizer; the organizer decides whether to disclose this information publicly. We find that concealing the information elicits higher expected total effort, regardless of the distribution of abilities. In addition, we find that the rent-dissipation rate of the contest does not depend on the disclosure policy.

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1. Introduction

One natural instrument for contest design is the choice to disclose or conceal information available to the contest organizer. Consider, for instance, an R&D procurement tournament. The buyer can form tentative judgments regarding participating firms' abilities to execute the project based on their proposals. On the job market, an employer can assess a candidate's ability from references. As another example, consider a crowdsourcing contest for a

computing code. The organizer learns coders' identities when they register, and can therefore infer their competence by their track records. In these contexts, the contest organizer knows more than individual participants about the profiles of their competitors. So does it pay for her to publicize the information?

We address this question in a multi-prize all-pay-auction setting. Following Moldovanu and Sela (2001, 2006) and Konrad and Kovenock (2010), bidders' abilities are measured by their marginal effort costs, which are randomly distributed. Each bidder's cost is privately known, but can be accessed by the contest organizer. The contest organizer strategically chooses between two policy alternatives: (1) fully revealing bidders' profiles publicly, versus (2) concealing them. Her disclosure policy is *ex ante* committed prior to the realization of bidders' cost profiles.

The timing of the game is as follows. First, the contest organizer commits to her disclosure policy publicly. Second, bidders' cost

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profiles are realized and learned by the organizer. This information is disclosed to all bidders if and only if the organizer has chosen full disclosure. Finally, bidders submit their effort entries simultaneously.

We show that concealing bidders' cost information elicits a higher *ex ante* expected total effort, regardless of their marginal cost distribution. However, bidders' expected payoffs and the contest's rent-dissipation rate do not depend on the prevailing disclosure policy.

When the contest organizer discloses bidders' cost information, a multi-prize complete-information all-pay auction arises. Clark and Riis (1998) characterize the bidding equilibrium for the case of multiple (homogeneous) prizes, while Siegel (2009) provides a closed-form formula for players' equilibrium payoffs in a general setting that allows for heterogeneous prizes and cost functions. The concealment policy leads to a multi-prize incomplete-information all-pay auction. For this setting, Moldovanu and Sela (2001, 2006) provide a solution to the bidding equilibrium. We use the techniques and results developed by these studies to compare the two disclosure policies.

This paper belongs to the literature on information revelation in contests. Lim and Matros (2009) and Fu et al. (2011) investigate an effort-maximizing contest organizer's incentive to disclose the number of actual participating bidders in Tullock contests. Aoyagi (2010) studies optimal feedback policy about agents' performance in a multi-stage tournament. Kovenock et al. (forthcoming) consider voluntary information sharing between two bidders on their values in an all-pay auction setting. Denter et al. (2011) study both players' strategic information transmission and mandatory disclosure policy in a two-player contest, in which one bidder's valuation is common knowledge, while the other's is privately known. In contrast, our study focuses on the contest organizer's incentive to disclose her information.

To our knowledge, our study is the first to investigate information disclosure in multi-prize contests. Our paper is closely related to that of Morath and Münster (2008), who compare two information structures (private independent values versus complete information) for standard auctions selling a single item, including all-pay auctions. They find that in all-pay auctions, bidders receive the same expected payoff across the two information structures, but a higher expected total effort results from a private-information setting. We generalize their findings by allowing for multiple prizes. A simultaneous study by Serena (2013), in a two-player winner-take-all Tullock contest, also examines the organizer's incentive to disclose information about contestants' abilities. In contrast, we focus on all-pay auctions and allow for multiple prizes.

2. A model of a multi-prize all-pay auction

Consider a multi-prize all-pay auction in which n bidders compete for m identical prizes, with $n \geq m \geq 1$. The value of each prize is normalized to 1. All prizes must be given away, but each bidder is eligible for only one prize. Bidders simultaneously exert their effort $x_i \geq 0$, and the m highest bidders are each awarded one prize. Ties are broken randomly.

A bidder i bears a marginal effort cost c_i , which is a random variable. The parameter c_i measures the bidder's ability, because a lower c_i implies more higher effort efficiency. All bidders are *ex ante* identical, such that c_i s are independently and identically distributed over a support $[c, \bar{c}] \in (0, +\infty)$, with a c.d.f $F(\cdot)$ and a p.d.f $f(\cdot)$. The realization of each c_i is observed by bidder i and the contest organizer, but not others. Its distribution, however, is commonly known.

The organizer decides whether to disclose the ability of contestants or to conceal the information and announces her choice

publicly, before c_i is realized. We denote disclosure by D and concealment by C . Nature then determines bidders' ability profiles $\mathbf{c} = (c_1, c_2, \dots, c_n)$. The organizer observes \mathbf{c} and discloses it if and only if she has committed to D . Bidders then simultaneously submit their effort entries $\mathbf{x} = (x_i)$.

2.1. Contests with full disclosure

We first consider the subgame in which policy D has been chosen. In this case, the contest organizer publicly discloses every bidder's marginal cost before bidders choose their efforts. A complete-information all-pay auction arises. Clark and Riis (1998) consider a setting in which bidders bear the same marginal effort cost, but value the prize differently. A simple transformation allows us to apply their results to our setting.³

Lemma 1 (Clark and Riis, 1998). For each $\mathbf{c} = (c_i)$, there exists a unique mixed-strategy equilibrium for the subgame, in which only the $m + 1$ most efficient bidders remain active, i.e., exerting positive effort. The expected effort of an active bidder i is $E(x_i) = p_i(m; \mathbf{c}) \frac{1}{c_i} - \left(\frac{1}{c_i} - \frac{1}{c_{(m+1)}}\right)$, where $p_i(m; \mathbf{c})$ is the equilibrium probability of his winning one of the m prizes, and $c_{(m+1)}$ denotes the $(m + 1)$ th lowest cost.

Lemma 1 presents a convenient formula for the *ex ante* expected total effort induced under policy D .

Lemma 2. Under policy D , the *ex ante* expected total effort induced is

$$R^D = \prod_{k=0}^m (n - k) \cdot \underbrace{\int_{c_1}^{\bar{c}} \int_{c_2}^{\bar{c}} \dots \int_{c_m}^{\bar{c}}}_{m+1} \left(\sum_{i=1}^{m+1} E(x_i) \right) [1 - F(c_{(m+1)})]^{n-m-1} \times \prod_{k=m+1}^1 dF(c_k). \tag{1}$$

Proof. Define $P_n^{m+1} = \binom{n}{m+1} (m + 1)!$, which is the number of the ordered sets of $m + 1$ bidders with the lowest costs. For each of these P_n^{m+1} symmetric cases, the total expected effort induced is identical. Therefore, the total expected effort must be P_n^{m+1} times the expected effort induced when the winners are bidder group $\{i | i = 1, 2, \dots, m + 1\}$ and their costs are ranked in ascending order, i.e., $c_1 < c_2 < \dots < c_m < c_{m+1} < \dots < c_n$:

$$R^D = P_n^{m+1} \times \underbrace{\int_{c_1}^{\bar{c}} \int_{c_2}^{\bar{c}} \dots \int_{c_m}^{\bar{c}}}_{m+1} \left[\left(\sum_{i=1}^{m+1} E(x_i) \right) \times \left(\underbrace{\int_{c_{m+1}}^{\bar{c}} \int_{c_{m+1}}^{\bar{c}} \dots \int_{c_{m+1}}^{\bar{c}}}_{n-m-1} \prod_{l=m+2}^n dF(c_l) \right) \right] \prod_{k=m+1}^1 dF(c_k)$$

³ Our model is strategically equivalent to that of Clark and Riis (1998) when, as in their setting, bidders' uniform marginal effort is normalized to one and bidder i values each prize for $1/c_i$. This equivalence has been well recognized in the literature, e.g., Wasser (2013).

$$\begin{aligned}
 &= P_n^{m+1} \underbrace{\int_{\underline{c}}^{\bar{c}} \int_{c_1}^{\bar{c}} \dots \int_{c_m}^{\bar{c}}}_{m+1} \left(\sum_{i=1}^{m+1} E(x_i) \right) [1 - F(c_{m+1})]^{n-m-1} \\
 &\quad \times \prod_{k=m+1}^1 dF(c_k) \\
 &= \prod_{k=0}^m (n-k) \\
 &\quad \cdot \underbrace{\int_{\underline{c}}^{\bar{c}} \int_{c_1}^{\bar{c}} \dots \int_{c_m}^{\bar{c}}}_{m+1} \left(\sum_{i=1}^{m+1} E(x_i) \right) [1 - F(c_{m+1})]^{n-m-1} \\
 &\quad \times \prod_{k=m+1}^1 dF(c_k). \quad \blacksquare
 \end{aligned}$$

2.2. Contests with concealment

We next analyze the subgame in which the contest organizer has committed to concealing the information about bidding costs (policy C). An incomplete-information all-pay auction arises.

The technique of Moldovanu and Sela (2006) can be adapted to search for a symmetric monotone pure-strategy equilibrium. We first introduce several notations used by Moldovanu and Sela. Let $F_{(i,n)}(\cdot)$ denote the c.d.f of the i th lowest order statistics of the set $\mathbf{c} = (c_1, c_2, \dots, c_n)$. By their Appendix A,

$$F_{(i,n)}(\cdot) = \sum_{j=i}^n \binom{n}{j} F^j(\cdot) [1 - F(\cdot)]^{n-j}, \tag{2}$$

and

$$F'_{(i,n)}(\cdot) = \frac{n!}{(i-1)(n-i)!} F^{i-1}(\cdot) [1 - F(\cdot)]^{n-i} F'(\cdot). \tag{3}$$

By their Corollary 1, we characterize the following symmetric equilibrium.

Lemma 3 (Moldovanu and Sela, 2006). *Under policy C, each bidder i has a symmetric equilibrium bidding strategy*

$$x(c_i) = \int_{c_i}^{\bar{c}} \frac{1}{c} F'_{(m,n-1)}(c) dc. \tag{4}$$

The contest elicits an ex ante expected total effort

$$R^C = m \int_{\underline{c}}^{\bar{c}} \frac{1}{c} dF_{(m+1,n)}(c). \tag{5}$$

3. Comparison between disclosure policies

We are now ready to compare the ex ante expected total efforts and bidders' payoffs across the two disclosure policies.

3.1. Effort comparison

We first compare the total effort that results from the two disclosure policies, i.e., R^D versus R^C .

Theorem 1. *Concealing bidders' cost information elicits higher ex ante expected total effort, i.e., $R^D < R^C$, regardless of the distribution of marginal effort cost c_i .*

Proof. Under policy C, by Eqs. (3) and (5), the ex ante expected total effort can be rewritten as

$$\begin{aligned}
 R^C &= m \int_{\underline{c}}^{\bar{c}} \frac{1}{c} dF_{(m+1,n)}(c) \\
 &= \frac{n(n-1) \dots (n-m)}{(m-1)!} \int_{\underline{c}}^{\bar{c}} \frac{1}{c} F^m(c) [1 - F(c)]^{n-m-1} dF(c).
 \end{aligned}$$

The rest of this proof proceeds in three steps.

Step 1 Let $i(k)$ denote the bidder with the k -th lowest cost for a given realization of \mathbf{c} . Under policy D – because every prize must go to one of the $m + 1$ active bidders with the $m + 1$ lowest costs – the following holds:

$$\sum_{k=1}^{m+1} p_{i(k)}(m; \mathbf{c}) = m, \quad \forall \mathbf{c} = (c_1, c_2, \dots, c_n). \tag{6}$$

Step 2 We claim that under policy D, for given \mathbf{c} , the equilibrium total expected effort is bounded from above by $\frac{m}{c_{(m+1)}}$.

By Lemma 1, the total expected effort is the sum of the $E(x_{i(k)})$ = $p_{i(k)}(m; \mathbf{c}) \frac{1}{c_{i(k)}} - \left(\frac{1}{c_{i(k)}} - \frac{1}{c_{(m+1)}} \right)$ for all $m + 1$ active players:

$$\begin{aligned}
 \sum_{k=1}^{m+1} E(x_{i(k)}) &= \sum_{k=1}^{m+1} \left[p_{i(k)}(m; \mathbf{c}) \frac{1}{c_{i(k)}} - \left(\frac{1}{c_{i(k)}} - \frac{1}{c_{(m+1)}} \right) \right] \\
 &= \sum_{k=1}^{m+1} \left[p_{i(k)}(m; \mathbf{c}) \frac{1}{c_{i(k)}} \right] - \sum_{k=1}^m \frac{1}{c_{i(k)}} + \frac{m}{c_{(m+1)}} \\
 &= \sum_{k=1}^m [p_{i(k)}(m; \mathbf{c}) - 1] \frac{1}{c_{i(k)}} \\
 &\quad + p_{i(m+1)}(m; \mathbf{c}) \frac{1}{c_{(m+1)}} + \frac{m}{c_{(m+1)}}.
 \end{aligned}$$

By (6), $p_{i(m+1)}(m; \mathbf{c}) = m - \sum_{k=1}^m p_{i(k)}(m; \mathbf{c})$. Therefore,

$$\begin{aligned}
 \sum_{k=1}^{m+1} E(x_{i(k)}) &= \sum_{k=1}^m [p_{i(k)}(m; \mathbf{c}) - 1] \frac{1}{c_{i(k)}} \\
 &\quad + p_{i(m+1)}(m; \mathbf{c}) \frac{1}{c_{(m+1)}} + \frac{m}{c_{(m+1)}} \\
 &= \sum_{k=1}^m [p_{i(k)}(m; \mathbf{c}) - 1] \frac{1}{c_{i(k)}} \\
 &\quad + \sum_{k=1}^m [1 - p_{i(k)}(m; \mathbf{c})] \frac{1}{c_{(m+1)}} + \frac{m}{c_{(m+1)}} \\
 &= \sum_{k=1}^m [p_{i(k)}(m; \mathbf{c}) - 1] \left[\frac{1}{c_{i(k)}} - \frac{1}{c_{(m+1)}} \right] + \frac{m}{c_{(m+1)}} \\
 &< \frac{m}{c_{(m+1)}} \text{ since } p_{i(k)}(m) - 1 \leq 0, \\
 &\quad c_{i(k)} < c_{(m+1)} \text{ for all } k \leq m.
 \end{aligned}$$

Step 3 We claim $R^D < R^C$.

The proof also extends from Lemma 2. Note that we use the notation of $\frac{m}{c_{m+1}}$ instead of $\frac{m}{c_{(m+1)}}$ to keep consistent with the proof of Lemma 2, where costs are already ranked in ascending order. We

have⁴

$$\begin{aligned}
 R^D &= \prod_{k=0}^m (n-k) \cdot \underbrace{\int_{\underline{c}}^{\bar{c}} \int_{c_1}^{\bar{c}} \cdots \int_{c_m}^{\bar{c}}}_{m+1} \left(\sum_{i=1}^{m+1} EX_i \right) [1 - F(c_{m+1})]^{n-m-1} \\
 &\quad \times \prod_{k=m+1}^1 dF(c_k) \\
 &< \prod_{k=0}^m (n-k) \cdot \underbrace{\int_{\underline{c}}^{\bar{c}} \int_{c_1}^{\bar{c}} \cdots \int_{c_m}^{\bar{c}}}_{m+1} \frac{m}{c_{m+1}} [1 - F(c_{m+1})]^{n-m-1} \\
 &\quad \times \prod_{k=m+1}^1 dF(c_k) \\
 &= \prod_{k=0}^m (n-k) \cdot \underbrace{\int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{c_{m+1}} \cdots \int_{\underline{c}}^{c_2}}_{m+1} \frac{m}{c_{m+1}} [1 - F(c_{m+1})]^{n-m-1} \\
 &\quad \times \prod_{k=1}^{m+1} dF(c_k) \\
 &= \prod_{k=0}^m (n-k) \\
 &\quad \times \int_{\underline{c}}^{\bar{c}} \frac{m [1 - F(c_{m+1})]^{n-m-1}}{c_{m+1}} \frac{1}{m!} F^m(c_{m+1}) dF(c_{m+1}) \\
 &= \frac{n(n-1) \cdots (n-m)}{(m-1)!} \\
 &\quad \times \int_{\underline{c}}^{\bar{c}} \frac{[1 - F(c_{m+1})]^{n-m-1}}{c_{m+1}} F^m(c_{m+1}) dF(c_{m+1}) \\
 &= R^C. \blacksquare
 \end{aligned}$$

Theorem 1 states that the contest organizer prefers concealment, as it elicits a higher *ex ante* expected total effort, regardless of the cost distribution. Full disclosure eliminates uncertainty in the contest. When choosing his effort, a bidder will be discouraged if he faces stronger opponents and slacks off if he faces weaker opponents. In contrast, under the concealment policy, bidders remain uninformed of their opponents' competence and must take into account every possibility when placing their bids. As bidders are *ex ante* identical, this uncertainty preserves an even playing field, which smooths effort supply in the contest.

3.2. Comparison on bidders' payoff and rent dissipation rate

To compare bidders' *ex ante* expected payoffs across the two policies, we first examine representative bidders' conditional expected payoffs for a given c_i under each policy.

We begin with the case of full disclosure. **Siegel's (2009) Theorem 1** leads to each bidder's expected payoff for given \mathbf{c} . Recall that $c_{(m+1)}$ denotes the $(m + 1)$ th lowest cost.

Lemma 4 (Siegel, 2009). For a given realization of profile \mathbf{c} , under policy *D*, a bidder i receives a positive expected payoff of $1 - \frac{c_i}{c_{(m+1)}}$ if and only if he is among the m most efficient bidders and zero otherwise.

Lemma 4 allows us to obtain a bidder i 's conditional expected payoff $\pi_i^D(c_i)$ for a given c_i while taking into account all possible realizations of \mathbf{c}_{-i} .

Corollary 1. For a given c_i , a bidder i 's conditional expected payoff is given by

$$\pi_i^D(c_i) = \int_{c_i}^{\bar{c}} \left[1 - \frac{c_i}{c} \right] dF_{(m,n-1)}(c). \tag{7}$$

We next consider the case of policy *C*, under which bidders' cost information is concealed. For a given c_i , a bidder's conditional expected payoff i is given by

$$\begin{aligned}
 \pi_i^C(c_i) &= 1 \cdot \Pr(c_i < c_{(m+1)}) - c_i x(c_i) \\
 &= 1 - F_{(m,n-1)}(c_i) - c_i x(c_i), \tag{8}
 \end{aligned}$$

by the equilibrium result of **Lemma 3**. Comparing $\pi_i^D(c_i)$ with $\pi_i^C(c_i)$ leads to the following.

Theorem 2. For a given c_i , the two policies render the same conditional expected payoff to each player, i.e. $\pi_i^D(c_i) = \pi_i^C(c_i)$.

Proof. By **Corollary 1**,

$$\begin{aligned}
 \pi_i^D(c_i) &= \int_{c_i}^{\bar{c}} \left[1 - \frac{c_i}{c} \right] dF_{(m,n-1)}(c) \\
 &= \int_{c_i}^{\bar{c}} dF_{(m,n-1)}(c) - c_i \int_{c_i}^{\bar{c}} \frac{1}{c} dF_{(m,n-1)}(c) \\
 &= F_{(m,n-1)}(\bar{c}) - F_{(m,n-1)}(c_i) - c_i \int_{c_i}^{\bar{c}} \frac{1}{c} dF_{(m,n-1)}(c) \\
 &= 1 - F_{(m,n-1)}(c_i) - c_i \int_{c_i}^{\bar{c}} \frac{1}{c} dF_{(m,n-1)}(c) \\
 &= 1 - F_{(m,n-1)}(c_i) - c_i x(c_i) \\
 &= \pi_i^C(c_i),
 \end{aligned}$$

where the second-to-the-bottom equality follows **Lemma 3**, Eq. (4). \blacksquare

By **Theorem 2**, it is straightforward to conclude that each bidder's *ex ante* expected payoff must be identical across the two policies, i.e., $E_{c_i} \pi_i^D(c_i) = E_{c_i} \pi_i^C(c_i)$. Bidders are indifferent to alternative disclosure policies.

The rent-dissipation rate of a contest is defined as (total effort cost)/(total prize value), which equals (total prize value – total expected payoff)/(total prize value). Therefore, the policies also induce the same rent-dissipation rate, since bidders' *ex ante* expected payoffs are the same. These implications are summarized below.

Corollary 2. Under the two disclosure policies, (i) bidders' *ex ante* expected payoffs are the same; and (ii) rent-dissipation rates are identical.

4. Conclusion

In this note, we find that concealing bidders' cost information renders higher expected efforts than disclosing it, regardless of their cost distribution. However, neither the rent-dissipation rate nor bidders' *ex ante* expected payoffs depend on the prevailing disclosure policy.

Our study, as well as the majority of the literature on information disclosure in contests, assumes *ex ante* identical players. It

⁴ The equality in lines 2-3 is derived by swapping integration order.

remains intriguing how results would vary when players are asymmetric. Denter et al. (2011) show that with one-sided asymmetric information, concealment could soften competition, which contrasts with our finding. A more systematic study is in demand to explore the ramifications of *ex ante* asymmetry in determining optimal disclosure policy. Further, we assume that the organizer *ex ante* commits to her disclosure policy. Both Lim and Matros (2009) and Denter et al. (2011) point out the nuance caused by the inability of commitment. It is interesting to explore the optimal disclosure policy in a multi-prize setting when the organizer's commitment power is limited.

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